24. $x^2y' = y - xy, \qquad y(1) = 2$

25. $y' = (e^{-x} - e^x)/(3 + 4y), \qquad y(0) = 1$

26. $2y\dfrac{dy}{dx} = \dfrac{x}{\sqrt{x^2 - 4}}, \qquad y(3) = -1$

27. $\sin 2x\,dx + \cos 3y\,dy = 0, \qquad y(\pi/2) = \pi/3$

28. $y^2(1 - x^2)^{1/2}dy = \arcsin x\,dx, \qquad y(0) = 1$

In Problems 29 through 36, obtain the requested results by solving the given equations analytically or, if necessary, by graphing numerically generated approximations to the solutions.

29. Solve the initial value problem

$$y' = (1 + 3x^2)/(12y^2 - 12y), \qquad y(0) = 2$$

and determine the interval in which the solution is valid.
Hint: To find the interval of definition, look for points where the integral curve has a vertical tangent.

30. Solve the initial value problem

$$y' = 2x^2/(2y^2 - 6), \qquad y(1) = 0$$

and determine the interval in which the solution is valid.
Hint: To find the interval of definition, look for points where the integral curve has a vertical tangent.

31. Solve the initial value problem

$$y' = 2y^2 + xy^2, \qquad y(0) = 1$$

and determine where the solution attains its minimum value.

32. Solve the initial value problem

$$y' = (6 - e^x)/(3 + 2y), \qquad y(0) = 0$$

and determine where the solution attains its maximum value.

33. Solve the initial value problem

$$y' = 2\cos 2x/(10 + 2y), \qquad y(0) = -1$$

and determine where the solution attains its maximum value.

34. Solve the initial value problem

$$y' = 2(1 + x)(1 + y^2), \qquad y(0) = 0$$

and determine where the solution attains its minimum value.

35. Consider the initial value problem

$$y' = ty(4 - y)/3, \qquad y(0) = y_0.$$

(a) Determine how the behavior of the solution as t increases depends on the initial value y_0.
(b) Suppose that $y_0 = 0.5$. Find the time T at which the solution first reaches the value 3.98.

36. Consider the initial value problem

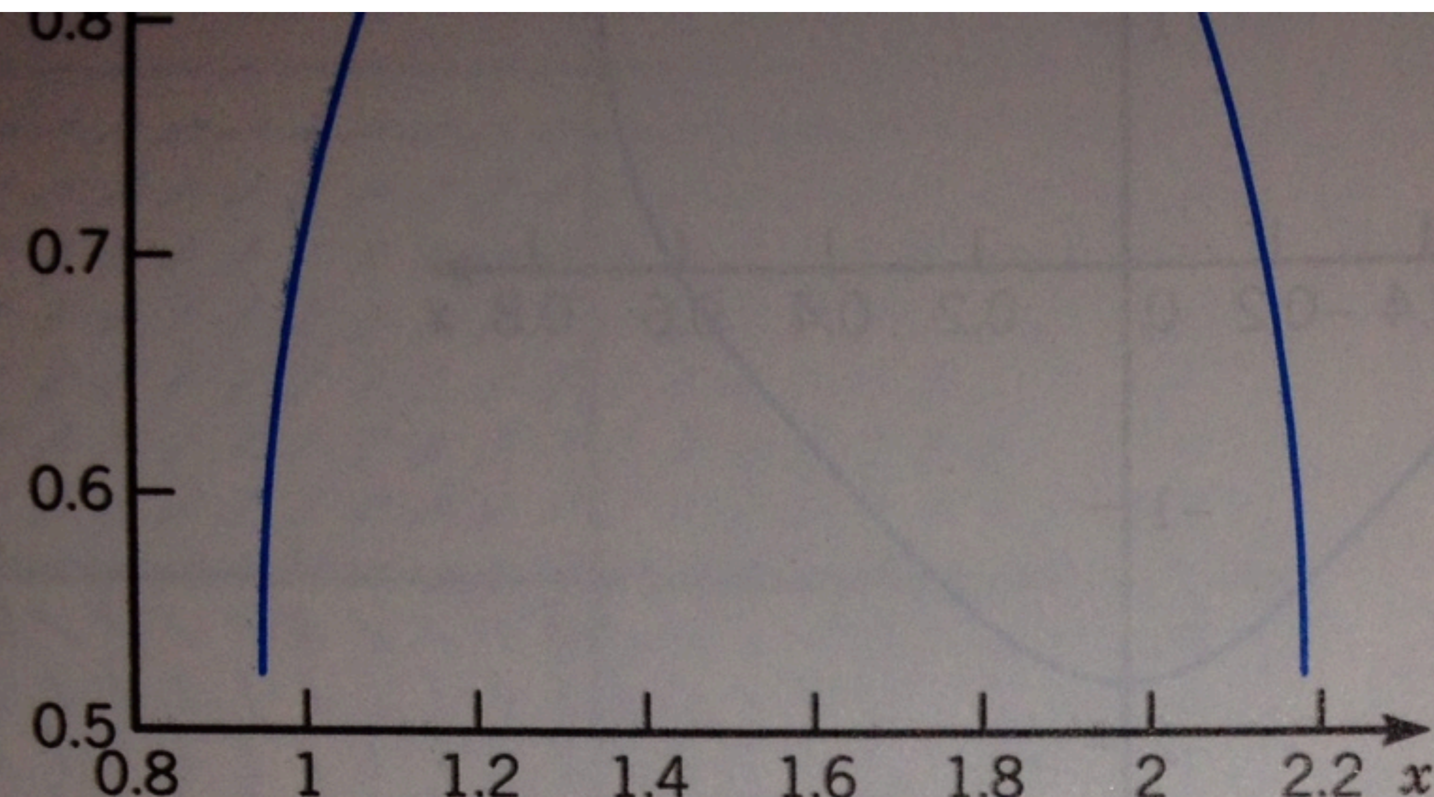
$$y' = ty(4 - y)/(1 + t), \qquad y(0) = y_0 > 0.$$

(a) Determine how the solution behaves as $t \to \infty$.
(b) If $y_0 = 2$, find the time T at which the solution first reaches the value 3.99.
(c) Find the range of initial values for which the solution lies in the interval $3.99 < y < 4.01$ by the time $t = 2$.

37. Solve the equation

$$\frac{dy}{dx} = \frac{ay + b}{cy + d},$$

where a, b, c, and d are constants.



(c) $|x - \pi/2| < 0.62$ approximately

29. $4y^3 - 6y^2 - x^3 - x - 8 = 0; -2 < x < \infty$

31. $y = -2/(x^2 + 4x - 2); x = -2$

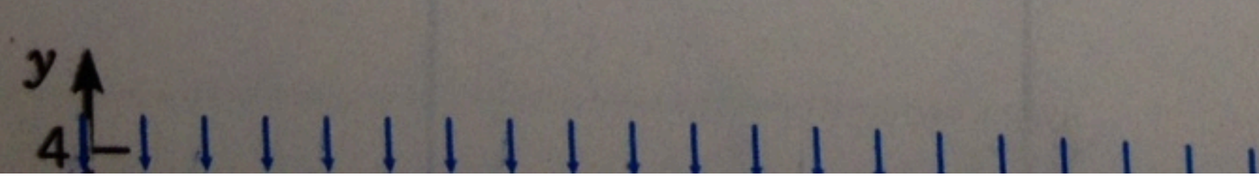
33. $y = -5 + \sqrt{16 + \sin 2x}; x = \pi/4 + n\pi$

35. (a) $y \to 4$ if $y_0 > 0$; $y = 0$ if $y_0 = 0$; $y \to -\infty$ if $y_0 < 0$
(b) $T \approx 3.29527$

37. $x = (c/a)y + (ad - bc)/a^2 \ln|ay + b| + k; a \neq 0,$
$ay + b \neq 0$

Section 2.2 Linear Equations: Method of Integrating Factors *page 54*

1. (a)



(b) some
crease wit

7. (a)