MATH 215A

Homework 1 – Due September 4, 2018

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Work on all of these problems, but carefully write up and turn in only problems 2, 3, 4, and 6.

Feel free (and encouraged!) to work with your classmates on this homework, but you **must** write up your own solutions. Indicate on your homework the set of people with whom you worked, if that set is non-empty, and cite any other sources you consulted besides your notes and your textbook.

(1) Read Chapter 0 of Hatcher, and do the following problems:

- pages 18–19: # 2, 3, 9, 14, 16, 17, 20.

(2) Show that $f : X \to Y$ is a homotopy equivalence if there exist maps $g, h : Y \to X$ such that $f \circ g \simeq id_Y$ and $h \circ f \simeq id_X$. More generally, show that f is a homotopy equivalence if $f \circ g$ and $h \circ f$ are homotopy equivalences.

(3) Show that $S^m * S^n \cong S^{m+n+1}$, where $*$ is the join operation (see Hatcher, page 9).

(4) Given a space X, show that the following are equivalent:

(a) X is contractible.

(b) Every map $f : X \to Y$ is null-homotopic, for any space Y.

(c) Every map $f : Y \to X$ is null-homotopic, for any space Y.

(5) Show that two spaces X and Y are homotopy equivalent if and only if for every space Z, there is a bijective correspondence

$$\phi_Z : [X, Z] \to [Y, Z]$$

such that for all continuous maps $h : Z \to Z'$, we have

$$h_* \circ \phi_Z = \phi_{Z'} \circ h_*,$$

where h_* is the map induced on homotopy classes of maps, that is, the following diagram commutes:

$$\begin{array}{ccc} [X, Z] & \xrightarrow{\phi_Z} & [Y, Z] \\ \downarrow h_* & & \downarrow h_* \\ [X, Z'] & \xrightarrow{\phi_{Z'}} & [Y, Z']. \end{array}$$

(6) Given CW complexes X and Y, explicitly describe the CW structure on $X \times Y$ in terms of the CW structures on X and Y. That means, describe the cells of $X \times Y$ and their attaching maps. Choose a CW structure for S^1, and work out the CW structure on the torus $S^1 \times S^1$ coming from the product CW structure.