Math 130
Homework 7 – Due November 1, 2016
Jamie Conway

1. Show that every isometry of $\mathbb{R}^2$ can be expressed as the composition of at most three reflections. You may wish to look at Stillwell, section 3.7, but you should give your own proof. In particular, you can use things we did in class.

2. Do the following problems from Stillwell: 7.1.1, 7.2.2 – 7.2.6

3. The *affine transformations* of $\mathbb{R}^2$ is the group of functions of the form $f(x) = Mx + v$, where $M \in GL_2(\mathbb{R})$ and $v \in \mathbb{R}^2$.

   (a) If $f_1(x) = M_1 x + v_1$ and $f_2(x) = M_2 x + v_2$, what is $f_1 \circ f_2$?

   (b) If $f_1$ is as above, what is its inverse?

   (c) Give examples of two concepts that are invariant under affine transformations.

4. Following section 7.6 of Stillwell, let $q = \cos(\theta/2) + i \sin(\theta/2)$.

   (a) Write $q$ as a $2 \times 2$ matrix.

   (b) Verify that the isometry $f_q : p \mapsto qpq^{-1}$ of $(i, j, k)$-space leaves the $i$-axis fixed and rotates the $(j, k)$-plane through an angle $\theta$.

5. (optional) Consider an *octahedron* in $\mathbb{R}^3$ with vertices $\pm(1, 0, 0)$, $\pm(0, 1, 0)$, and $\pm(0, 0, 1)$. How many elements are in the group of rotations of the octahedron? What quaternions make up the rotations of the octahedron? Justify your assertions. You might also want to look at section 7.7 of Stillwell.

6. (optional – not to hand in) Why is there no orientation-preserving isometry of the sphere $S^2$ that does not fix any points? Can you prove this by imitating our strategies from class? Or do you need to use a "three reflections theorem"? You might find it helpful to do the exercises at the end of section 7.4 to start.