

Math 130
Homework 6 – Due October 20, 2016
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1. In class, we defined the *order* of a finite projective plane to be one less than the number of points on each line. Show that this number is well-defined, *i.e.* that each line has the same number of points. *Hint: given two lines ℓ_1 and ℓ_2 , find a bijection $\ell_1 \rightarrow \ell_2$.*
2. If we have a geometry G , then define the *dual geometry* $G!$ such that the set of points of $G!$ are the set of lines of G , and the lines of $G!$ are the points of G . A point of $G!$ is on a line of $G!$ if the corresponding line of G contains the corresponding point of G .
 - (a) If G is a projective plane, show that $G!$ also satisfies the projective plane axioms. *Hint: use the result from HW5 that every point of G has at least 3 lines passing through it.*
 - (b) A consequence of (a) is that every theorem in projective geometry has a *dual* theorem. What is the dual theorem to question 1 above?
 - (c) (not to hand in) If G is a projective plane, when is $G!$ isomorphic to G ? Here, isomorphic means that there's a bijection $f : \text{points}(G) \rightarrow \text{points}(G!)$ such that the lines of G get mapped to the lines of $G!$.
3. Using question 1 and its dual (from 2(b)), show that every finite projective plane of order n has exactly $n^2 + n + 1$ points. By duality, your proof will also show that there are exactly $n^2 + n + 1$ lines.
4. Do the following problems from Stillwell: 5.6.1 – 5.6.4.

Do the set of fractional linear transformations form a group? Explain.
5. Let $[p, q; r, s] = \frac{(p-r)(q-s)}{(q-r)(p-s)}$.
 - (a) Show that $[p, q; r, s] \cdot [p, q; s, r] = 1$.
 - (b) For which other permutations of the inputs is $[p, q; r, s] \cdot [?, ?, ?, ?] = 1$? Fill in the ?s with p, q, r , and s .
 - (c) What is the relationship between $[p, q; r, s]$ and $[p, r; q, s]$?
 - (d) For which other permutations does the relationship in (c) hold?