Math 130

Homework 4 – Due September 27, 2016

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1. Consider a geometry where the points are ordered pairs (a, b) of *rational numbers*, and the lines are all the subsets of the form

$$\{t(x_0, y_0) + (x_1, y_1) \,\big|\, t \in \mathbb{Q}\},$$

where (x_i, y_i) are fixed points in $\mathbb{Q} \times \mathbb{Q}$, and $(x_0, y_0) \neq (0, 0)$.

Prove that this satisfies axioms (I1-3). Which of the betweenness axioms (B1-4) are satisfied? Use the same betweenness definition that we used for $\mathbb{R}^2$ in class on Thursday.

2. Can you replace $\mathbb{Q}$ with $\mathbb{Q}(\sqrt{2})$ in Problem (1)? What about any other subfield of $\mathbb{R}$?

To answer this question, you need to give a clear explanation of whether your previous work used special properties of $\mathbb{Q}$, or if every property of $\mathbb{Q}$ that you used also work for other subfields of $\mathbb{R}$. You don't need to re-prove everything.

3. Do you following problems from Hartshorne (pages 71–73 and 79–80):

2.6.3(a) (and draw a picture for a 7-point projective plane), 2.6.10 (hint: induction)

2.7.1, 2.7.4, 2.7.9, 2.7.10

4. Read (and learn) the proof of Plane Separation, proposition 2.7.1 in Hartshorne. (nothing to hand in)

5. Verify that the projective plane that we introduced in class satisfies the axioms (P1-4) for a projective plane given in Hartshorne problem 2.6.3. Recall that the points are $\mathbb{R}^2 \cup [0, \pi)$, and the lines are all of the form $\{(x, y), \theta \,\big|\, y = mx + b \text{ and } \theta = \arctan m\}$ or $\{(a, y), \pi/2\}$ (vertical lines), or $\{\theta \,\big|\, 0 \leq \theta < \pi\}$ (all the slopes together).