Math 53 Spring 2016
Quiz 2 2/8/16 Name: **Solutions**

You have 15 minutes to take this quiz. Do not use notes, calculators, or classmates to assist you.

1. Consider the points $P = (2, 4, 3), Q = (4, -2, 1)$.

a) Find an equation of the plane through the origin, P, and Q.

b) Find an equation of the plane through P, Q, and the point $R = (1, 1, 1)$.

a) Call the origin O. The plane is spanned by the vectors $\overrightarrow{OP} = \langle 2, 4, 3\rangle$ and $\overrightarrow{OQ} = \langle 4, -2, 1\rangle$, so a normal vector is

$$\langle 2, 4, 3\rangle \times \langle 4, -2, 1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & 3 \\ 4 & -2 & 1 \end{vmatrix} = \langle 10, 10, -20\rangle$$

and an equation of the plane is

$$10(x - 0) + 10(y - 0) - 20(z - 0) = 0$$

which we can simplify to $10x + 10y - 20z = 0$, or just $x + y - 2z = 0$. (This is why the question asked for *an* equation of the plane.)

b) One option would be to use the same method as in (a), but with R instead of O. An easier method is to observe that, since $1 + 1 - 2 = 0$, R lies in the plane of part (a), so $x + y - 2z = 0$ is also the answer to (b).

2. Consider the plane with equation $2x - y + 4z = 3$ and the line with equation $-x = y/2 = z$. Explain why they are parallel. (Hint: Find vectors that describe them and use a vector operation.)

Solution We can parametrize the line as $(-t, 2t, t)$, which shows that it has direction vector $\langle -1, 2, 1\rangle$. Similarly, the plane has normal vector $\langle 2, -1, 4\rangle$.
The plane and the line being parallel is equivalent to the direction vector of the line being perpendicular to the normal vector of the plane, and to check this we compute the dot product:

$$\langle -1, 2, 1\rangle \cdot \langle 2, -1, 4\rangle = -2 - 2 + 4 = 0$$

The dot product is zero, so the vectors are perpendicular as required.