Worksheet #28: Midterm 3 Review
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Math 53: Fall 2022
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Problem 1. Find the volume of the torus $\rho = \sin\phi$. Torus means "donut-shaped"; for the given torus, how big is the "donut hole"?

Problem 2. Let C be the ellipse $x^2/4 + y^2 = 1$, oriented counterclockwise. Explain why Green's Theorem cannot be directly used to evaluate the line integral

$$\oint_C (y \log_4(x^2 + 4y^2) + 3x^2y^2\cos(x^3))\, dx + (2y\sin(x^3))\, dy.$$

Find a way around this issue and evaluate the integral anyways. A hint is in this footnote.[1]

Problem 3. Let S be some region in the uv-plane with area 10; I'm not going to describe exactly what shape S is. Consider the change of variables

$$\begin{aligned} x &= 8u + 9v \\ y &= 11u + 12v \end{aligned}$$

Applying this change of variables transforms the region S to a new region T in the xy-plane.

(a) Prove that this change of variables is one-to-one (aka bijective) by giving an inverse change of variables.

(b) What is the area of T?

Problem 4. Let D be the disk of radius a centered at the origin. What's the average distance of a point on D from the origin?

Problem 5. Convert the following triple integral to Cartesian coordinates. You do not need to evaluate it.

$$\int_0^{\pi/2} \int_0^1 \int_{r^2}^1 r^2 \cos\theta \, dz\, dr\, d\theta.$$

Problem 6. Suppose a surface in 3D space is given by a level set $g(z, r, \theta) = 0$ in cylindrical coordinates. Derive the following formula for the normal vector to this surface:

$$\vec{n} = \left\langle g_r \cos\theta - \frac{1}{r} g_\theta \sin\theta,\ g_r \sin\theta + \frac{1}{r} g_\theta \cos\theta,\ g_z \right\rangle.$$

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[1] Break the line integral into two parts, one of which is a line integral of a conservative vector field and the other of which is easy to do directly.