Worksheet #26: Whose Line Integral Is It Anyway?
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Problem 1. Sketch the following vector fields, and then determine whether or not the given vector field is conservative.

If you think the vector field is conservative:

- Find a potential function for it.
- Compute the path integral of this vector field between the points $(1, 1)$ and $(3, 2)$.

If you think that the vector field is not conservative:

- Prove it by showing that the vector field isn't irrotational.
- Using your sketch, draw a loop over which the path integral looks nonzero.

(a) $F(x, y) = \langle x, 0\rangle$

(b) $F(x, y) = \langle xy, xy\rangle$

(c) $F(x, y) = \langle \sin(y), \cos(x)\rangle$

(d) $F(x, y) = \frac{1}{x^2+y^2}\langle x, y\rangle$

Problem 2. Suppose that a cable has constant linear density k and has the shape of the helix

$$x = 4\cos t, y = 4\sin t, z = 3t, 0 \leq t \leq \pi/2.$$

Find its center of mass.

Problem 3.

(a) The vector field $F = \langle 3x^2yz - 3y, x^3z - 3x, x^3y + 2z\rangle$ is conservative. Find a potential function for it.

(b) Let C be the line segment starting at $(0, 0, 2)$ and ending at $(0, 3, 0)$. Evaluate

$$\int_C \langle 3x^2yz - 3y + e^{\pi \arctan(yz)^2}, x^3z - 3x + 2\cos(x^2), x^3y + 2z\rangle \cdot dr.$$

(Hint: break the integral into two integrals, and use part (a) for one of these.)