Worksheet #24: Everybody Loves the Jacobian
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Problem 1. Give a conceptual explanation of why the surface area form is

$$\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dA$$

in the case of a surface of the form $z = f(x, y)$.

Problem 2. Describe in words what the change of variables

$$x = u\cos\theta - v\sin\theta$$
$$y = u\sin\theta + v\cos\theta$$

does. (Hint: set $u = r\cos\varphi$ and $v = r\sin\varphi$ and apply some trig identities. Drawing a picture might help.) Compute the Jacobian of this transformation and explain why your answer is reasonable.

Problem 3. (Stewart Exercise 15.9.22) By applying an appropriate change of variables, compute the area enclosed by the four curves $xy = a, xy = b, xy^{7/5} = c$, and $xy^{7/5} = d$, where $0 < a < b$ and $0 < c < d$, and contained in the first quadrant. This computation is important in thermodynamics, since this area represents the work done by an ideal Carnot engine.