

Worksheet #22: Thrice is Nice

Date: 10/26/2022

Math 53: Fall 2022

Instructor: Norman Sheu

Section Leader: CJ Dowd

Problem 1. Consider the 3D region enclosed by the parabolic cylinder $y = x^2$ and the planes $z = 0$ and $y + z = 1$. Sketch this region (you saw it in lecture on Tuesday). Then set up *six* different integrals to compute the volume of this region corresponding to the six possible orders of integration (i.e. the six permutations of dx, dy, dz). No need to evaluate these.

Problem 2. Compute the volume of the region lying inside the sphere $x^2 + y^2 + z^2 = 4$ but outside the cylinder $x^2 + y^2 = 1$ using cylindrical coordinates in two different ways: once in the $dr dz d\theta$ order, and once in the $dz dr d\theta$ order. (Why haven't I asked you to do all six this time?) Try to sketch this region.