Worksheet #21: This is getting out of hand! Now, there are two of them!
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Problem 1. Let R be the rectangle with corners $(0,0),(2,0),(2,3),(0,3)$. Suppose that $f(x,y)$ is a function such that $f_x > 0$ and $f_y < 0$ on R. If we want to use a Riemann sum to overestimate the integral $\int\int_R f(x,y)\;dx\,dy$, where we break up R into six 1×1 squares, where should we pick the sample points?

Problem 2. You are asked to integrate some function $f(x,y)$ over some region R in the plane. In each of the following scenarios, say whether you are inclined to use the integration order $dx\,dy$ or the integration order $dy\,dx$. How strongly do you feel about your chosen order, i.e. is the other order still potentially worth trying? Except for part (d), set up a double integral with your chosen order. If you think both orders are plausible, set up a double integral for both orders.

(a) The region R is the triangle with corners $(0,0),(1,1),(1,-1)$.

(b) The region R is the region defined by $x^2 \le y \le x$.

(c) The region R is the bounded region between the two curves $x = y - y^3$ and $x = y^2 - 1$.

(d) The function is $f(x,y) = e^{x^2}$.

Problem 3. The value of the integral

$$\int\int_R (3 - x^2 + 2x - 4y^2)\;dx\;dy$$

depends on the region R chosen. (It might even be undefined if R is unbounded.) What choice of region R maximizes the value of this integral?