Worksheet #20: Prepare for Trouble//And Make It Double
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Problem 1. Let D be the region in the first quadrant that lies between the circles $x^2 + y^2 = 1$ nad $x^2 + y^2 = 2$. Sketch this region, and evaluate

$$\int\int_D x \, dA.$$

(Polar coordinates is probably the way to go.) Determine the center of mass of D, assuming it has uniform density.

Problem 2. Sketch the region $0 \leq x \leq 1, x^2 \leq y \leq 1$ in the plane. Evaluate the double integral

$$\int_0^1 \int_{x^2}^1 \sqrt{y} \sin y \, dy \, dx.$$

(Hint: Reverse the order of integration.)

Problem 3. Find the moment of inertia of:

(a) A square with side length a and constant mass density ρ, rotating about its center.

(b) A disk with radius 1 and constant mass density ρ, rotating in 3D space about its diameter. (You'll probably need to use a reverse trig substitution. You can think of this as the MOI associated to flipping a coin.)