Worksheet #19: Midterm 2 Review, Part 2
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Math 53: Fall 2022
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Problem 1. For each of the following properties, either give an example of a continuously twice-differentiable function $f(x, y) : \mathbb{R}^2 \to \mathbb{R}$ satisfying the property or prove that no such function exists.

(a) $f_{xx}f_{yy} - (f_{xy})^2 = 0$ at $(x, y) = (0, 0)$ and f has a local maximum at $(0, 0)$.

(b) $f_{xy} = 0$ at $(0, 0)$ and f has a saddle point at $(0, 0)$.

(c) f has no maximum when rescricted to the unit circle $x^2 + y^2 = 1$.

(d) f has infinitely many critical points when restricted to the unit circle $x^2 + y^2 = 1$.

Problem 2. The curve parametrized by $r(t) = \langle t, \sqrt{\frac{3}{2}}t^2, t^3\rangle, -\infty < t < \infty$ is a variant of the twisted cubic.

(a) Find the length of the portion of $r(t)$ that lies inside the sphere $x^2 + y^2 + z^2 = 7/2$.

(b) When $r(t)$ intersects this sphere, what angle does the tangent vector $r'(t)$ make with the normal vector to the sphere?

Problem 3. Find a function $f(x, y, z) : \mathbb{R}^3 \to \mathbb{R}$ such that

$$\nabla f = \langle 3x^2yz - 3y, x^3z - 3x, x^3y + 2z\rangle.$$

Problem 4. Prove that the following system of equations has at least two solutions (x, y, λ):

$$\begin{aligned} ye^{yx} &= 4x^3\lambda \\ xe^{xy} &= 6y^5\lambda \\ x^4 + y^6 &= 2 \end{aligned}$$

(You do not need to find these solutions.)

Problem 5. Determine the point(s) on the ellipsoid $\frac{x^2}{16} + \frac{y^2}{9} + (z - 1)^2 = 1$ that is/are furthest from the origin.