Worksheet #18: Midterm 2 Review, Part 1
Date: 10/12/2022
Math 53: Fall 2022
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I am ill at these numbers.

—Polonius, *Hamlet*, Act II, Scene 2.

Yo, Lagrange multipliers are sick.

—All of you, now, probably.

Problem 1.

(a) Determine whether $\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{(x+1)(x^2+xy+y^2)}$ exists, and if it does, find its value.

(b) Describe the domain of the function $f(x,y) = \frac{\sqrt{xy}}{x}$. Determine whether $\lim_{(x,y)\to(0,0)} f(x,y)$ exists, and if it does, find its value.

Problem 2. Describe the critical points of $f(x,y,z) = e^{-(xyz)^2}$. Determine if this function has a global minimum; if so, find it and describe where it is attained. Do the same for a global maximum.

Problem 3. Suppose that a function $f(x,y,z)$ of three variables only depends on the length of its input; that is, there exists some function $g : \mathbb{R}^{\geq 0} \to \mathbb{R}$ such that $f(x,y,z) = g(\|r\|)$ where $\vec{r} = (x,y,z)$. Such f is called a *radial* function.

Find an expression for the gradient $\nabla f(x,y,z)$ in terms of g and r. This means that your final answer should not include any of the symbols f, x, y, or z.

Problem 4. A woman is climbing a very steep slope, described by the plane $f(x,y) = y$. To make the climb easier, she decides to walk it in a switchback. That is, instead of taking the steepest possible ascent, which result in an incline of 45 degrees from going in the direction $(0,1)$, she walks in some oblique direction $\vec{u} = (a,b)$ for a shallower angle of incline. There are two directions in the (x,y) plane that she can walk to give an incline of 10 degrees. What are these directions?

Problem 5. Find the minimum and maximum values of $f(x,y) = 8x^2 - 2y$ subject to the constraint $x^2 + y^2 \leq 1$.