Worksheet #16: Going Supercritical
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Math 53: Fall 2022
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Problem 1. Let $f(x, y) : \mathbb{R}^2 \to \mathbb{R}$ be a differentiable function of two variables. True or false:

1. Every local extremum (minimum or maximum) of f occurs at a critical point.
2. If f has a critical point at P and $f_{xx}(P)$, $f_{yy}(P)$, and $f_{xy}(P)$ are all positive, then f has a local minimum at P.
3. If f has a local maximum at P, then $f_{xx}(P)$ and $f_{yy}(P)$ must both be negative.

Problem 2. Let $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ be the function that measures distance from the origin. Compute $D_{\vec{u}} f(1, 2, 3)$ for the three following values of $\vec{u}$:

$$\frac{1}{\sqrt{14}}\langle 1, 2, 3\rangle, \langle -1, -1, 1\rangle, \text{ and } \langle 1, 2, 3\rangle + \langle -1, -1, 1\rangle.$$

Problem 3. (Stewart Exercise 14.7.60.) Find an equation

$$a(x - 1) + b(y - 2) + c(z - 3) = 0$$

of the plane that contains the point $(1, 2, 3)$ and cuts the smallest possible volume off of the corner of the first octant.

- It may be helpful to know that the volume V of a simplex (a four-sided pyramid whose faces are all triangles) with vertices $(0, 0, 0), (x, 0, 0), (0, y, 0)$, and $(0, 0, z)$ is $V = \frac{xyz}{6}$.
- To check your answer, the correct minimum volume is 27.
- I think this quite a difficult problem, though it is doable with the tools available to you from lecture. Do it only if you've finished everything else. If you want to fully finish the problem within a reasonable amount of computation, then you'll need to be smart about simplifying where you can.