Worksheet #13: Partial Credit
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Problem 1. What goes wrong if you try to generalize the definition

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

to define the derivative of a function f of more than one variable?[1]

There are a number of notations for a partial derivative. If $f(x, y)$ is a function of two variables, then f_x, $\frac{\partial f}{\partial x}$ and D_x all mean the same thing. You should get used to all of them.

Problem 2. Let $f(x, y)$ and $g(u, v)$ be two functions related by

$$g(u, v) = f(e^u + \sin v, e^u + \cos v).$$

Use the following values to calculate $g_u(0, 0)$ and $g_v(0, 0)$ (not all of these will be relevant):

$f(0,0) = 3$	$g(0,0) = 6$	$f_x(0,0) = 4$	$f_y(0,0) = 8$
$f(1,2) = 6$	$g(1,2) = 3$	$f_x(1,2) = 2$	$f_y(1,2) = 5.$

(Hint: this is an application of the multivariate chain rule. How is $g_u = \frac{\partial}{\partial u} f(e^u \sin v, e^u + \cos v)$ related to f_x and f_y?)

Problem 3. Compute $f_{yxyxx}(2, 0)$ if $f(x, y) = \frac{7}{8}(y^2 + y + x)^4$. (The notation f_{yxyxx} means the same thing as $\frac{\partial^5 f}{\partial x^3 \partial y^2}$. Remember that you can choose to do the partial derivatives in any order, and some orders are easier than others.)

[1] There is a good way to think about *the* derivative of a multivariable function using a slight modification of this definition, usually called the *total derivative* to distinguish it from the partials. In the case $f : \mathbb{R}^n \to \mathbb{R}$, the total derivative the same thing as the gradient.