Worksheet #12: Limit Break
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Math 53: Fall 2022
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Problem 1.

(a) Explain the difference between these two statements:

(i) The limit $\lim_{(x,y)\to(a,b)} f(x,y)$ exists.

(ii) The limits of $f(x,y)$ along any line through (a,b) all exist and are equal.

(b) Sketch a 3D graph of the piecewise defined function

$$f(x,y) = \begin{cases} 1: \text{ if } x^2 = y \text{ and } x \neq 0 \\ 0: \text{ else.} \end{cases}$$

(This might feel very weird.) Setting $(a,b) = (0,0)$, show that statement (i) is false for f but statement (ii) is true.

Problem 2. Suppose f is a function whose domain is $D = \{(0,0)\} \cup \{(x,y) : |x^2 + y^2| \geq 1\}$. (Ask me or your groupmates about this notation if it is new to you.) Sketch the domain D in the plane. Is f continuous at $(0,0)$?[1]

Problem 3. Suppose that S is a (sufficiently nice and smooth) surface in $\mathbb{R}^3$ that contains the two curves

$$r_1(t) = (2 + 3t, 1 - t^2, 3 - 4t^2)$$
$$r_2(u) = (1 + u^2, 2u^3 - 1, 2u + 1).$$

Using this information, compute an equation of the tangent plane to S at the point $(2,1,3)$. (What do you have to do to find the normal vector of this plane?)

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[1] You can do this without using any other information about f. If this exercise confuses you, I'd suggest looking at Definition 1 in Stewart 14.2 very carefully.