Worksheet #10: I'm Running Out of Ideas for Amusing Vector-Themed Worksheet Titles
Date: 09/23/2022
Math 53: Fall 2022
Instructor: Norman Sheu
Section Leader: CJ Dowd

Problem 1. A particle starts at position $r(0) = (3, 1, 2)$ and has velocity given by $r'(t) = (2t, 0, e^t)$. What is its position at time $t = 3$?

Problem 2. What's the difference between a vector function and a parametric curve?

Problem 3. Prove that taking derivatives of a cross/dot product of vector functions $u(t), v(t)$ obeys the "product rule":

(a) $\frac{d}{dt}[u(t) \cdot v(t)] = u'(t) \cdot v(t) + u(t) \cdot v'(t)$

(b) $\frac{d}{dt}[u(t) \times v(t)] = u'(t) \times v(t) + u(t) \times v'(t)$

Use the the first of these to show that if $r(t)$ has constant speed (explain what this means!), then $r''(t)$ is orthogonal to $r'(t)$.

Problem 4. For a vector-valued function $r : \mathbb{R} \to \mathbb{R}^3$, give an interpretation of the three vector functions $T(t) = r'(t)/\|r'(t)\|$, $N(t) = T'(t)/\|T'(t)\|$, and $T(t) \times N(t)$. These are called the unit tangent, normal, and binormal vectors respectively. Prove that they are mutually orthogonal and of unit length (we call a set of vectors with this property "orthonormal").