

Worksheet #9: Midterm 1 Review

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Math 53: Fall 2022

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These questions are somewhat more difficult than what I would expect of the real midterm. You will also have more time per question on the actual midterm. If you can answer all of these with only minor algebra mistakes, you should be in excellent shape.

Problem 1. Write down a polar equation for the line $y = \sqrt{2}(1 - x)$ of the form $r = f(\theta)$. (Hint: Trying to substitute $x = r \cos \theta, y = r \sin \theta$ doesn't seem to work very well. Draw a graph of the line. What do you get if you rotate the graph of this line 45 degrees about the origin?)

Problem 2. What is the rightmost point of the curve defined by parametric equations $x = 2t - t^2, y = (t - 1)^3, -\infty < t < \infty$? What is equation of the tangent line at this point? (Bonus: What does the slope of the tangent line tell you about whether the graph is "smooth" or "pointy" here?)

Problem 3. Sketch the limaçon $r = \frac{1}{2} + \cos \theta$ and find the area between the outer loop and the inner loop.

Problem 4.

- (a) Find the angle between a diagonal of a cube and one of its edges.
- (b) Find a vector perpendicular to both a diagonal and a side of the cube.

Problem 5. Show that the planes $2x - 3y + z = 4$ and $4x - 6y + 2z = 3$ are parallel and find the distance between them.