

**Worksheet #8: Crucifixus**

**Date: 09/16/2022**

**Math 53: Fall 2022**

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**Problem 1.** Are there any vectors  $v \in \mathbb{R}^3$  such that  $(1, 2, 1) \times v = (3, 1, -5)$ ? If so, find all of them; otherwise, prove that none exist. Do the same question for  $(1, 2, 1) \times v = (3, 1, 5)$ .

**Problem 2.** Suppose I am riding a monorail in a straight line in the direction  $\vec{v}$ . There is a wind blowing with constant force  $\vec{F}$  on the monorail. The monorail is sturdy; pushing it in a direction perpendicular to the rail does nothing. With this in mind, what is the *effective* force on the monorail, i.e. the component of the wind force that will actually do anything? How is this situation related to projection and the dot product? What does it have to do with work, in the physics sense? (This type of scenario will become very important not too long from now when we talk about path integrals.)

**Problem 3.** Let  $L_1$  be the line passing through the points  $(1, -2, 4)$  and  $(2, 1, 3)$ , and let  $L_2$  be the line passing through  $(0, 3, -3)$  and  $(2, 4, 1)$ .

- (a) Write parametric equations for each of these lines.
- (b) Are  $L_1$  and  $L_2$  parallel, skew, or intersecting? If they intersect, where? If they do not intersect, how far apart are they, and where are they closest, i.e. which pair of points  $P_1 \in L_1$  and  $P_2 \in L_2$  minimizes the distance  $d(P_1, P_2)$ ?