Worksheet #7: Fisiks
Date: 09/12/2022
Math 53: Fall 2022
Instructor: Norman Sheu
Section Leader: CJ Dowd

Problem 1. In 3D space, I shoot a cannonball out of a cannon at the speed of light ($= 3 \times 10^8\ m/s$) from the origin. I can describe the direction the cannon is pointing using two angles: the angle θ describes which direction in the xy-plane the cannon is pointing, and the angle φ describes the angle of elevation of the cannon off the ground.

(a) Draw (i) bird's eye view of the cannon in the xy-plane, labeling the angle θ and (ii) a diagram of the cannon from the side, labeling the angle φ.

(b) What is the velocity vector of the cannonball the moment it gets shot? Check your answer by making sure the length of this vector is equal to the speed of light.

(c) Acceleration due to gravity is $10\ m/s^2$. Write down the acceleration vector of the cannonball, ignoring air resistance. Then write down the velocity vector as a function of time t after the cannon gets shot, and then finally write down the position vector of the cannonball as a function of t. (You can assume that t is small enough that the cannonball hasn't hit the ground yet).

Problem 2. (Expressing reflection with vectors.)

(a) A beam of light travelling with velocity v bounces off a flat mirror with normal vector n. We can write

$$v = \frac{v \cdot n}{\|n\|^2} n + \left(v - \frac{v \cdot n}{\|n\|^2} n \right).$$

Explain why this is the decomposition of v into its *normal* and *tangent components* relative to the mirror. (If the concept of *projecting* a vector onto another vector is new to you, talk it over with your groupmates!)

(b) Use this decomposition and the fact that reflection is a *linear transformation*[1] to show that the new velocity of the light after reflecting is

$$v - 2\frac{v \cdot n}{\|n\|^2} n.$$

Check to make sure that the speed hasn't changed, i.e. the length of this vector is equal to $\|v\|$.

(c) In 2D space, a beam of light travels in the vertical straight line $x = x_0$ for some $-1 < x_0 < 1$, going northward at a speed of 1. However, the unit circle has been replaced by a perfectly circular mirror. The beam of light hits this circular mirror and reflects off in a new direction. Draw a picture of the situtation and then find the velocity vector of the light beam after reflecting off the mirror; this velocity vector will depend on x_0. Sanity check your answer in the cases $x_0 = 0, \pm\sqrt{2}/2$ and in the limits as $x_0 \to \pm 1$.

---

[1] A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is a function that takes a vector as input and gives a vector as output and satisfies the property $T(\lambda v + \mu w) = \lambda T(v) + \mu T(w)$ for any vectors $v, w \in \mathbb{R}^n$ and scalars λ, μ.