Worksheet #5: The Heroic Age of Polar Exploration
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Math 53: Fall 2022
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Problem 1. By now you've seen plane curves described in a number of different ways (as functions $y = f(x)$, as solutions to more general Cartesian equations, as parametric curves, as polar curves), and often you are asked to graph these curves. One of the most common questions I get asked is how one should go about graphing a curve by hand. Sometimes it's hard to see the forest through the trees.

(Discussion.) Why do we care about graphing curves? If you're given equations (rectangular, parametric, polar, etc) for a curve that you've never seen before, how should you go about graphing it by hand? What should your graph use as guidelines, and what should it emphasize?

Problem 2. Give an informal explanation why the length form is $ds = \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\, d\theta$ in polar coordinates. In other words, why should we expect this to be the right formula for the infinitesimal arc length assocated to an infinitesimal change in θ?

Problem 3. (Based on Stewart, Example 3 in Chapter 10.4)

(a) Sketch graphs of the two curves $r = \cos 2\theta$ and $r = \frac{1}{2}$ on top of one another. (Hint: the curve $r = \cos 2\theta$ is the "four-leaf rose," which you saw in lecture yesterday.)

(b) Find all points of intersection between these two curves. (Why do you get more than you might expect from just solving algebraically?)

(c) Find the area of the intersection of the regions enclosed by these two curves. (Hint: symmetry. You'll still need to compute the area of two separate pieces and add them up. Justify your bounds of integration.)