

Problem 1. Determine polar equations for the following curves:

- (a) The circle of radius a centered at the origin.
- (b) The vertical line $x = 1$.
- (c) The line $x = y$.
- (d) The parabola $x^2 = y$.

Problem 2. Recall our favorite parametric curve, the astroid $x = \cos^3 t, y = \sin^3 t$. Since $r = \sqrt{x^2 + y^2}$, you might think that polar coordinates for the astroid are given by

$$r = \sqrt{\cos^6 \theta + \sin^6 \theta}$$

but this is in fact *false*. Why is this incorrect?

Problem 3. When we have rectangular coordinates for a curve, it's relatively easy to shift the curve vertically or horizontally: to shift the curve h units to the right and k units upward, just substitute $x - h$ for x and $y - k$ for y . In parametric coordinates $(x, y) = (f(t), g(t))$, this shift is given by $(x, y) = (f(t) + h, g(t) + k)$. Think about why it is, in general, difficult to describe a translation in polar coordinates. If you want something more concrete to chew on, try coming up with polar coordinates for the parabola $y = x^2 + 1$ and see how messy it gets compared to your answer to 1(d).

Problem 4. Derive the formula for the slope of a line tangent to a polar curve $r = f(\theta)$:

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}.$$

(Hint: start by writing $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ using the chain rule, and then use the formulas for x and y in terms of r and θ .) This is an example of a formula I'd recommend knowing how to re-derive if necessary rather than memorizing it.