Worksheet #3: Hearts and Diamonds
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Math 53: Fall 2022
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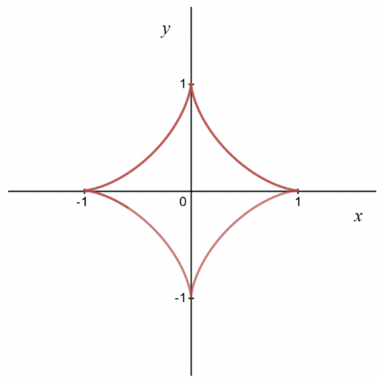
Problem 1. True or false: if $x(t)$ and $y(t)$ are twice-differentiable and $x''(t) \neq 0$, then

$$\frac{d^2y}{dx^2} = \frac{d^2y/dt^2}{d^2x/dt^2}.$$

Problem 2.

(a) Express the length of the curve $(x(t), y(t)), t \in [a, b]$ as an integral, i.e. give a general formula for computing arc length of a parametric curve.

(b) (Stewart 10.2.54.) Compute the arc length of the curve $(x, y) = (\cos^3(t), \sin^3(t)), t \in [0, 2\pi]$. This shape is known as an *astroid*, and it looks like this:



Problem 3. Recall from lecture that a *cycloid* is curve that can be described parametrically as the motion of a point on the circumference of a wheel rolling along the ground. A *cardioid* is what you get if the wheel rolls along another circle instead of the ground.

(a) Suppose that both circles have radius a, and their centers start at $(-a, 0)$ and $(a, 0)$ as shown in the diagram (so the parametric curve starts at the origin). Derive parametric equations for the cardioid. (Hint: think about the outer circle's rotation in a vacuum. It rotates at a constant rate; how does this rate compare to its rate of revolution about the inner circle?)

(b) (Messy, optional.) Prove that the arc length of the cardioid is $16a$.