Worksheet #2: Call the parametrics
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Problem 1. For many parametric curves $(x, y) = (f(t), g(t))$, we cannot express y as a function of x or vice versa; give an example. Despite this, the derivative $\frac{dy}{dx}$ at a given point $(x, y) = (a, b)$ on the curve usually makes sense if f, g are differentiable unless $\frac{dx}{dt} = f'(x) = 0$ at $x = a$. Discuss how this is possible.

Problem 2. An ant is crawling on a table. Its position at time t is given by $(x, y) = (f(t), g(t))$. What is the ant's speed at any given time?

Problem 3. For a parametric curve $(x, y) = (f(t), g(t))$, $t \in \mathbb{R}$, describe another parameterization that gives the same curve. And another. And another...

Problem 4. The *cuspidal cubic* is the curve $\{x^3 = y^2 : x, y \in \mathbb{R}\}$, i.e. the set of points (x, y) for which $x^3 = y^2$ and x and y are real numbers.

(a) Draw the cuspidal cubic. (Hint: take a square root.) Is y a function of x?

(b) Come up with functions $x = f(t), y = g(t)$ to parameterize the cuspidal cubic. (You might want to try part (b) first if you are struggling with part (a).)

(c) Does $\frac{dy}{dx}$ exist at $(x, y) = (0, 0)$? What about $\frac{dx}{dy}$? How is this reflected visually in the graph?

Problem 5. Let $(x, y) = (f(t), g(t))$ be a parametric curve. Is it possible for $\frac{dy}{dx}$ to exist even at a point where $\frac{dx}{dt} = 0$?