Math 220, Spring 2012, homework 7, due Wednesday March 14

Page 80, no. 15; page 104, nos. 1,2,3, page 105, nos. 4,7 (hint for no. 7: use no. 4).

Consider the SODE $du = dt + dw$, where w is BM and u^0 is a given constant, discretized as $u^{n+1} = u^n + k + \eta$, $\eta \sim N(0, k)$, where k is a time step, with observations $b^{n+1} = 2u^{n+1} + \tau$, $\tau \sim N(0, s)$. Find $P(u^{n+1}|u^n)$, $P(b^{n+1}|u^{n+1})$, and F (minus the log of the product of the last two probabilities). Find $\phi = minF$, set up the equation $F(u^{n+1}) - \phi = \xi^2/2$, where $\xi \sim N(0, 1)$. Find a solution formula such that the map $\xi \to u$ is one-to-one and onto, and find the sampling weights. Describe the full algorithm for estimating $P(u^{1:n}|b^{1:n})$.