Math 220, Spring 2012, homework 5, due Wednesday February 29

Consider the heat equation $v_t = (1/2)v_{xx}$ inside the region $D : t > 0, 0 \leq x \leq 1$, with data $v(x,0) = \phi(x)$ for $0 \leq x \leq 1$, and $v(0,t) = v(1,t) = 0$ for $t > 0$. Show that the solution at a point $(x_0, t_0)$ inside $D$ is $v(x_0, t_0) = \int F[w(\omega, \cdot)]dW$, where $\int dW$ is a Wiener integral, and the functional $F$ assigns to each BM defined on $[0, t_0]$ the number 0 if the curve $x = x_0 + w(\omega, s), t = t_0 - s$, $(0 \leq s \leq t_0)$, crosses either of the lines $x = 0$ or $x = 1$ before it reaches the $x$-axis, and $F = \phi(x_0 + w(\omega, t_0))$ otherwise. (This is an updated version of problem 2, page 78.)

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