Math 128a, Chorin, Spring 2013, homework 8, due the week of April 1.

1. For each one of the schemes below for solving $y' = f(x, y), y(0) = a$, discuss how many initial conditions are needed to start the calculation, check the root condition, find an expression for the truncation error, and discuss whether the scheme is stable, consistent, or convergent: (i) $u_{i+1} = u_{i-1} + h(f_{i-1} + f_{i+1})$; (ii)$u_{i+1} = u_{i-1} + h(f_{i-1} + f_i + f_{i+1})$; (iii)$u_{i+1} = u_i + hf_{i+1}$. ($f_i$ means $f(ih, u_i)$).

2. Consider the scheme $u_{i+1} = u_{i-1} + 2hf_i$ for solving $y' = -y, y(0) = 1$ (f_i is defined as above). Find the value u_1 by the Euler scheme. Find the solution of the difference equations in the form $u_i = C_1\rho_1^i + C_2\rho_2^i$, using the data to determine C_1, C_2. Check that the scheme is stable. Show that one of ρ_1, ρ_2 is negative and one is positive, and that the one that is negative has a larger absolute value than the other one. This explains what you observe in computer HW no. 4.

3. Consider a difference scheme of the form $u_{i+1} = au_i + bu_{i-1}$, where a, b are constants. There are two solutions of the form ρ_1^i and either ρ_2^i if $\rho_1 \neq \rho_2$ or $i\rho_1^i$ otherwise. Check that these solutions satisfy the difference equation. Show that they are linearly independent using the definition of linear independence. Observe that if u_0, u_1 are given, the scheme defines $u_2, u_3, \ldots$ uniquely. Deduce that all solutions of the scheme are linear combinations of the two you found.