Math 128a, Chorin, Spring 2013, homework 6, due the week of March 11.

1. Find polynomials H_0, H_1, H_2 (H_i of degree i) orthonormal with respect to the inner product

$$(f,g)_w = \int_{-\infty}^{+\infty} f(x)g(x)w(x)dx,$$

where $w(x) = e^{-x^2/2}/\sqrt{2\pi}$. (It may be useful to note that $\int_{-\infty}^{+\infty} w(x)dx = 1$.) These polynomials are known as "Hermite" polynomials.

2. Show that H_n has n distinct real roots.

3. Obtain an integration rule for evaluating the integral $\int_{-\infty}^{+\infty} f(x)w(x)dx$ ($w(x)$ defined as above) by interpolating $f(x)$ (NOT $f(x)w(x)$!) at n points $x_1, x_2, \ldots, x_n$ by a polynomial P_{n-1} of degree $n-1$ and deriving a formula of the form $\int f(x)w(x) = \sum a_i f(x_i)$ (what you need is a formula for the coefficients a_i; it's OK to leave the formula as an integral, you don't have to evaluate it).

4. Show that if the interpolation points are the zeroes of H_n the integration rule can be made exact for functions f which are polynomials of degree up to $2n-1$.

5. Find the zeroes of H_2 and show that if you use these points as interpolation points, the requirement that the integration rule be exact for polynomials of degree ≤ 3 gives you 4 equations for 2 coefficients, which are compatible and can be solved. Solve them and get the coefficients.