Math 128a, Chorin, Spring 2013, homework 5, due the week of March 4.

1. (i) Find coefficients a_{-1}, a_0, a_1 so that $f''(x) = a_{-1}f(x-h) + a_0 f(x) + a_1 f(x+h) + O(h^2)$, where f'' is a second derivative. (ii) Find coefficients $a_{-2}, a_{-1}, a_0, a_1, a_2$ so that $f'' = \sum_{i=-2}^{i=2} a_i f(x+ih) + O(h^4)$. (Hint: one easy way of doing this is showing that the differentation rule in part (i) can be extrapolated, and then rewriting the extrapolation as a single formula).

2. The mid-point rule approximates the integral $I = \int_a^b f(x)dx$ by the sum $I_h = \sum_{i=0}^{n-1} hf(x_{i+1/2})$, where $x_{i+1/2} = a + h/2 + ih$ for $i = 0, 1, \ldots, n-1$ and $h = (b-a)/n$. Show that there exists a constant C independent of h such that $I = I_h + Ch^2 + O(h^4)$ (so that extrapolation is possible).

3. Check that you get the correct formula $f'(x) \sim (f(x+h) - f(x-h))/(2h)$ by finding the polynomial P_2 that interpolates f at the points $x-h, x, x+h$ and then differentiating P_2 at x. (Be careful about the arguments of the functions: when you are finding P_2 the points $x-h, x, x+h$ are held fixed.)

4. Derive a Newton-Cotes integration formula with an error $O(h^6)$ (this error estimate should hold without doing an extrapolation). This can be a messy calculation. To make your life easier note: (i) by an argument discussed in class, it is enough to find a formula that has an error $O(h^5)$, you get one order for free, but you have to explain why and how this is true; and (ii) you only have to calculate in detail two of the coefficients, you can find the others either by consistency or by symmetry, but you have to explain what you are doing.