Math 128a, Chorin, Spring 2013, homework 4, due the week of February 25.

1. Find the zeroes, the minima and the maxima of the Chebyshev polynomial $t_4(x) = \cos(4 \arccos(x))/2^3$ in $[-1, +1]$, and plot the graph of this polynomial.

2. Suppose you define an inner product by the formula $(f, g)_w = \int_{-1}^{1} w(x)f(x)g(x)dx$, where w is a given function (the subscript w is there to distinguish this inner product from the usual inner product).
(i) Show that $\sqrt{(f, f)_w}$ is not a norm if w is zero at all points of a subinterval of $[-1, 1]$. You can assume (correctly) that $\sqrt{(f, f)_w}$ is a norm if $w > 0$. Call this norm $||\cdot||_w$.
(ii) Show that the Chebyshev polynomials are orthogonal in the inner product $(\cdot, \cdot)_w$ when $w = 1/\sqrt{1 - x^2}$.
(iii) With w as in (ii), how does one modify the method of finding a best approximation in the $||\cdot||_2$ norm when one wants to get a best approximation in the $||\cdot||_w$ norm?

3. Suppose you approximate the function $f(x) = e^{-x}$ in $[-1, 1]$ by interpolating it at the roots of T_{10} (the tenth Chebyshev polynomial). Find a bound on the infinity norm of the error in this approximation (note that you asked only for this bound, not to find the polynomial itself, which is a messy calculation).

4. Using equidistant interpolation points, find a difference formula that approximates f' with an error $O(h^4)$.

5. Suppose the values of the function f, known to be in C^7, are available only at the points $x, x+h/2, x+h, x+2h$. How accurately can one approximate $f'(x)$ (i.e., if the error is $O(h^k)$ for some k, what is the highest k one can get)?