Math 128a, Chorin, Spring 2013, homework 12, due the week of April 29.

1. (This is the example I did on the board). Consider the pair of equations $0.000100\, x_1 + 1.00\, x_2 = 1.00$, $1.00\, x_1 + 1.00\, x_2 = 1.00$, solved on a computer working in base 10, with 3 digits stored and with rounding. The true solution is $x_1 = 1.00010$, $x_2 = 0.99990$, which on this computer would appear as $1, 1$. Find the solution the computer would get (i) with partial pivoting, and (ii) without pivoting.

2. Consider the matrix A with rows (1, .99), (.99, .98). Find $||A||_2$, $||A^{-1}||_2$, and $cond(A)$. Calculate Ax, Ay, where $x = (1,\ 1)$, and $y = (3,\ -1.0203)$. Conclude that $\delta b = (-000097,\ +000106)$ yields $\delta x = (2,\ -2.0203)$, so that

$$||\delta x||/||x|| \sim 40000||\delta b||/||b||,$$

($\sim$ means "approximately"). Check that $40000 \sim cond(A)$. To see what is going on, note that you are looking for the intersection of nearly identical lines, and that the location of this intersection changes a lot when the slopes of the lines change by a small amount.

   It may be useful to know that the inverse of the matrix with rows (a, b), (c, d) is is $1/(ad - bc)$ times the matrix with rows $(d, -b)$, $(-c, a)$ provided $ad - bc \neq 0$. Also, if β is small, you can estimate $\sqrt{1 + \beta}$ by $(1 + \beta/2)$.