Math 128a, Chorin, Spring 2013, homework 10, due the week of April 15.

1. Produce a stable ODE solver with a local truncation error $O(h^6)$. (Hint: look up the homeworks about higher-order quadrature rules). You can assume that any algebraic equations that may appear will be solved exactly.

2. Suppose the corrector in a predictor-corrector methods is $u_{i+1} = u_{i-1} + (h/3)(f_{i-1} + 4f_i + f(u_{i+1}, u^*_{i+1}))$, where $f_i = f(ih, u_i)$. Find the order of the local truncation error of the corrector (i.e., the n in the statement "the local truncation error in the corrector with the equation for u_{i+1} solved exactly is $O(h^n)$"). Create a predictor such that the local truncation error of the resulting predictor-corrector pair is of the same order as that of the corrector standing alone. There is nothing to stop you from using values of u_j with $j < i - 1$ if you wish to do so.

3. Find the order of the local truncation error for the the predictor-corrector pair $u^*_{i+1} = u_i + hf_i$, $u_{i+1} = u_{i-1} + (h/3)(f_{i-1} + 4f_i + f(x_{i+1}, u^*_{i+1}))$.

4. Write out the components of the the scheme $u_{i+1} = u_i + (h/2)(f_i + f_{i+1})$, where $f_i = f(ih, u_i)$, for the case where y is a vector $y = (y_1, y_2, y_3)$ and $f(x, y) = (y_1^2 + 2y_1y_2\, , y_2 - y_1 + x\, , y_3x)$.