Math 128a, Chorin, Spring 2013, homework 1, due the week of February 4.

1. Consider a computer where the base is 10, the number of digits in the floating point representation is 3, and the exponent is between -1 and +2. What is the "machine δ", i.e., the bound of ϵ in the estimate $|fl(x) - x| \leq \epsilon|x|$? What is the largest number than can be represented? what is the smallest non-zero positive number than can be represented?

2. Suppose that a polynomial $P_2(x)$ of degree 2 has a simple root at x_1 and a double root at x_2. Show that it must be identically 0 (i.e., it is simply the number 0). (Reminder: at the double root $P_2' = 0$).

3. Show that a polynomial $P_n(x)$ of degree n that vanishes (i.e., equals 0) at $n + 1$ distinct points $x_0, x_1, x_2, \ldots x_n$ must be identically zero as follows: write the equations $P_n(x_i) = 0$ for $i = 0, 1, 2, \ldots, n$ as a $(n + 1)$by$(n + 1)$ linear system of algebraic equations with a zero right-hand side, and show that the determinant of this system is different from zero.

4. Does the order in which the multiplications are carried out when one calculates the product xyz matter as far as round-off error is concerned? (explain your answer).

5. For each of the following functions decide for what values of the initial guess x_1 Newton's method for finding roots converges, and for each x_1 that starts a convergent iteration, say which solution is produced: (i)$f = x^2 - 2x + 1$; (ii)$f = sgn(x)\sqrt{|x|}$, where $sgn(x) = 1$ for $x \geq 0$ and $sgn(x) = -1$ for $x < 0$.; (iii) $f = tanh(x)$.