

Qualifying Exam Syllabus

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July 18 2019, 10am-1pm, 939 Evans

Committee: David Nadler (Exam Chair), Ian Agol (Advisor), Richard Bamler, Holger Müller (Physics)

1 Major topic: Algebraic Topology (Geometry and Topology)

References: Hatcher, *Algebraic Topology*, Chapter 0, 1.1-1.3, 2.1-2.3, 3.1-3.3, 4.1-4.3; Milnor, Stasheff, *Characteristic Classes*, Section 1-15.

- **CW complexes.** Operations on spaces; Homotopy type.
- **Fundamental Group.** Van Kampen Theorem; Covering spaces.
- **Homology.** Simplicial homology; Singular homology; Cellular homology; Excision; Long exact sequence of pair; Mayer-Vietoris sequence.
- **Cohomology.** Universal Coefficient Theorem; Cup product; Kunneth Formula; Cap product; Poincare Duality.
- **Homotopy Theory.** Whitehead Theorem; Cellular approximation; CW approximation; Homotopy excision; Hurewicz Theorem; Fibrations; Obstruction theory.
- **Characteristic Classes.** Stiefel-Whitney classes; Euler class; Chern classes; Pontrjagin classes.

2 Major topic: Riemannian Geometry (Geometry and Topology)

Reference: do Carmo, *Riemannian Geometry*, Chapter 1-10.

- **Riemannian Manifolds.** Smooth manifolds; Maps between smooth manifolds; Vector fields; Riemannian metrics.
- **Connections.** Affine connections; Levi-Civita connection.
- **Geodesics.** Exponential map; Normal neighborhood.
- **Curvature.** Curvature; Sectional curvature; Ricci curvature; Scalar curvature; Space forms.
- **Completeness.** Hopf-Rinow Theorem; Cartan-Hadamard Theorem.
- **Variations.** Jacobi fields; Conjugate points; First and second variations of energy; Bonnet-Myers Theorem; Synge-Weinstein Theorem.
- **Comparison Theorems.** Index lemma; Rauch Theorem.

3 Minor topic: Lie Groups, Lie Algebras, Representation Theory (Algebra)

Reference: Humphreys, *Introduction to Lie Algebras and Representation Theory*, Chapter 1-24.

- **Definitions.** Lie groups; Lie algebras.
- **Semisimple Lie algebras.** Killing form; Cartan's criterion; Complete reducibility; Cartan and Borel subalgebras; Root space decomposition; Automorphisms.
- **Root systems.** Weyl group; Dynkin diagram; Classification.
- **Representation Theory.** Weights; Multiplicity formulas; Characters.