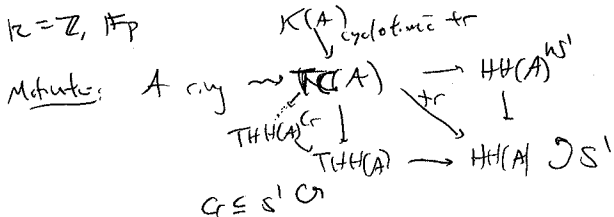


Typical HH + WH vectors (Mats. Speirs)

$k = \mathbb{Z}, \mathbb{F}_p$



HH recall  $A$  (dg) ~~alg~~ algebra /  $k$ .

$$HH(A) = \begin{matrix} A & \xrightarrow{L} & A \\ A \otimes A & & \end{matrix}$$

Thm  $(HHK(A))$   $A$   $k$ -algebra morph

$$HH_*(A) \cong \Omega_{A/k}^{-}$$

Def  $A$  ring spectrum

$$THH(A) = \begin{matrix} A & \xrightarrow{L} & A \\ A \otimes A & & \end{matrix}$$

Spectrum

Def A spectrum  $E$  is a sequence  $E_0, E_1, \dots$

- / map  $\Sigma E_n \rightarrow E_{n+1}$

analogy:

classical alg	spectral alg
Set	$\mathcal{S}p$ Top / sSet
$Ab$	Spectra
$(\mathbb{Z}, \mathbb{Z})$	$(\mathcal{A}, \mathcal{S})$

$\mathcal{S}p$  smash prod,  $\mathcal{S}p$  have spect.

Eilenberg-MacLane spectra

$$H: Ab \rightarrow \mathcal{S}p.$$

$$A \mapsto HA = (K(A, 0), K(A, 1), \dots)$$

has left adjoint  $\Pi_0$   
 $\Pi_0(E) = \text{colim } \Pi_n E_n$

(on connective  $\mathcal{S}p$ )

! fully faithful functor

Def  $\pi_*(E) = \text{colim}_R \pi_{n+2k}(E_k)$  (even if  $n < 0$ ).

Spectra has - lax monoidal structure.  $(HA \wedge HB \rightarrow H(A \wedge B))$   
 $\bullet$  A ring structure means  $HA$  also monoidal in  $\mathcal{S}_p$

$\pi_* \mathcal{S} = \text{stable homotopy gr + spheres}$  )  $\mathcal{S} \neq H\mathbb{Z}!$   
 $= \mathbb{Z}, \sigma_1, \sigma_2, \sigma_4, \sigma_8, \dots$  (but is with  $\otimes \mathbb{Q}$ .)

$\pi_* HA = A, 0, 0, \dots$

$H\mathbb{F}_p \wedge H\mathbb{F}_p \neq H(\mathbb{F}_p \otimes \mathbb{F}_p)$  in fact  $\pi_*(H\mathbb{F}_p \wedge H\mathbb{F}_p)$   
lax monoidal dual Steenrod alg.

Another def. A ring spectrum ( $= HA$ )

$$\text{THH}(A) = \left| (C_n) \longmapsto A^{\wedge^{L^{-1}/\partial \Delta^{L-1}}} \right|$$

$$= \left| A \rightleftharpoons A \wedge A \rightleftharpoons A \wedge A \wedge A \dots \left| \right.$$

Thm (Cisinski)  $\text{THH}(A)$  has  $S$ -action

Thm (Bokstedt periodicity)

$$\text{THH}(H\mathbb{F}_p) = H\mathbb{F}_p[S] \quad |S| = 2$$

$$\text{THH}(\mathbb{Z}) =$$

$\text{THH}(A)$  has special type +  $S$ -action: cyclic.

$X$  top space Consider  $\mathcal{L}X$  Maps  $(S^1, X) \mathcal{G} S^1$

$C_r \in S^1$  finite cyclic subgp.

$$(\mathcal{L}X)^{C_r} \xrightarrow[\text{equiv.}]{} \mathcal{L}X$$

$$\bigcup_{S^1/C_r} \xrightarrow{\cong} S^1$$

Def A cyclic spectrum  $T$  is a  $S^1$ -spectrum w/ maps  $\forall C_r \subseteq S^1$

$$\Phi_{C_r}^T \xrightarrow{\cong} T$$

"geometric fixed pts"  $\Phi^{G_r}: Sp^G \rightarrow Sp$  manifold  $\Phi^{G_r}(\sum_n X) = \sum_n X^{G_r}$

+ compatibility.

Other fixed pts:  $E$  a spectrum,  $E = (E_0, E_1, \dots)$   $E^{G_r} = (E_0^{G_r}, E_1^{G_r}, \dots)$

Fact This is a map

$$(-)^{G_r} \longrightarrow \Phi^{G_r}$$

The (Boklandt, Hring, Madsen) THH(A) admits cyclic structure

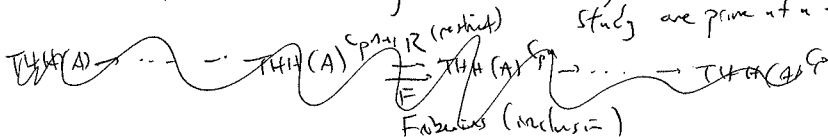
Consequences:

Restriction maps:  $T$  cyclic

$$T^{C_r} = (T^{C_r})^{C_r} \longrightarrow (\Phi^{C_r} T)^{C_r} \xrightarrow{\cong} T^{C_r}$$

with restriction map

THH(A), THH(A)<sup>C\_r</sup> from diagram. (e.g. fix p prime. Study are prime at a tensor)



transfer map  $\vee \begin{pmatrix} \text{TTH}(A) \mathbb{C}^n \\ \mathbb{R} \end{pmatrix} \downarrow \mathbb{F} = \text{Fubinius (inclusion)} \\ \text{TTH}(A) \mathbb{C}^n$

Witt vectors ( $p$ -typical)

$W^{(p)} = \mathbb{C}\text{-ring} \rightarrow \mathbb{R}\text{-ring}$

$W^{(p)}(A) = A^{\mathbb{N}}$  w/ exotic ring structure.

e.g.  $W^{(p)}(\mathbb{F}_p) = \mathbb{Z}_p$

Finite length Witt vectors

$W_n^{(p)}(A) = A^{\mathbb{N} \times \{1, \dots, n\}}$  w/ <sup>exotic</sup> ring structure.

$$\begin{array}{ccc} W_n(A) \cong (a_0, \dots, a_n) & (0, a_0, \dots, a_{n-1}) \\ \vee \left( \mathbb{R} \right) \downarrow \mathbb{F} = \text{Fubinius} & \mathbb{R} \downarrow \\ W_n(A) & (a_0, \dots, a_{n-1}) \end{array} \quad \begin{array}{c} \uparrow \vee \\ \downarrow \end{array}$$

Thm (Hasseholt Maser)

$A$  comm ring,  $\pi_0 \text{TTH}(A) \mathbb{C}^n = W_{n+1}^{(p)}(A)$

$$\begin{pmatrix} W = \mathbb{Z}_p \otimes W_n \\ \text{TTH} = \mathbb{Z}_p \otimes \text{TTH} \mathbb{C}^n \end{pmatrix}$$

Thm (4)

$A$   $\mathbb{F}_p$ -alg, smooth

$$\pi_n \text{TTH}(A) \mathbb{C}^n \cong W_n \Omega_{A/\mathbb{F}_p}^1[\delta]$$