Least-squares
March 4, 2016
Math 53

$$\sum d_i^2 = \sum (y_i - (mx_i + b))^2$$

$$= \sum y_i^2 - 2y_i(mx_i+b) + (mx_i+b)^2$$

$$(mx_i+b)^2 = m^2x_i^2 + 2mbx_i + b^2$$

$$= \sum y_i^2 - 2mx_iy_i - 2y_ib + m^2x_i^2 + 2mbx_i + b^2$$

Data points: $(2,6), (2, \frac{13}{2}), (3,7), (7,8)$ (inches, 5ft, shoe size)

$$f(m,b) = (6^2 + (\tfrac{13}{2})^2 + 7^2 + 8^2) - 2m(12+13+21+56) - 2b(6+\tfrac{13}{2}+7+8)$$
$$+ m^2(4+4+9+49) + 2mb(2+2+3+7) + 4b^2$$

$$= 191.25 - 2m \cdot 102 - 2b(27.5) + m^2(66)$$
$$+ 2mb(14) + 4b^2$$

$$f(m,b) = 191.25 - 204m - 55b + 66m^2 + 28mb + 4b^2$$

$$\frac{\partial f}{\partial m} = -204 + 132m + 28b = 0 \Rightarrow m = +1.55 - .21b$$

$$\frac{\partial f}{\partial b} = -55 + 28m + 8b = 0 \Rightarrow 8b = +55 - 43.4 + 5.88b$$

$$+28(+1.55 - .21b) = 43.4 - 5.88b$$

$$\Rightarrow 2.12b = +11.6$$

$$\Rightarrow b = +5.47$$

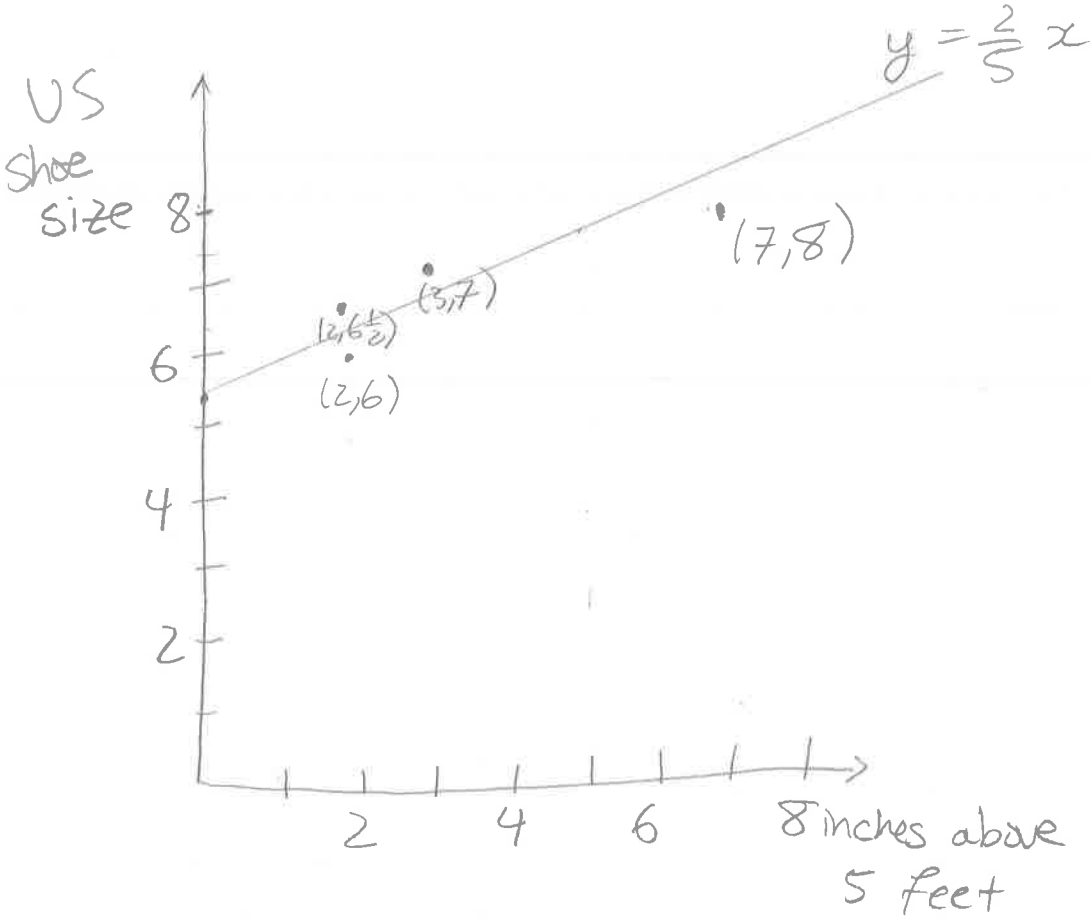
$$m = .40130$$

Best fit line $\boxed{y = .4x + 5.47}$

Min? $f_{mm}f_{bb} - (f_{mb})^2 = (132)(8) - (28)^2 = 272 > 0$

$f_{mm} = 132 > 0 \Rightarrow$ minimum

OVER



$$y = \frac{2}{5}x + 5.47$$