Practice Problems: line integrals of a vector-valued function

1) Find $\int_C xy\, dx + (x-y)\, dy$, where C = line segments from $(0,0)$ to $(2,0)$ and $(2,0)$ to $(3,2)$.

$C_1: \vec{r}_1(t) = \langle 2t, 0 \rangle, \ 0 \le t \le 1$, $\quad C_2: \vec{r}_2(t) = (1-t)\langle 2,0\rangle + t\langle 3,2\rangle$

$= \langle 2+t, 2t \rangle, \ 0 \le t \le 1$

$\int_{C_1} (2t)(0)\cdot 2dt + (2t-0)\cdot 0$

$= 0$

$\int_0^1 (2+t)(2t)dt + (2+t-2t)\, 2dt$

$= \cancel{4t} + 2t^2 + 4 + 2t - \cancel{4t}$

$= \int_0^1 2t^2 + 2t + 4\, dt = \frac{2}{3} + 1 + 4 = 5\frac{2}{3}$

$= \boxed{\frac{17}{3}}$

$\int_{C_1} + \int_{C_2} = 0 + 17/3 = \boxed{17/3}$

2) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = x^2y^3\, \hat{\imath} - y\sqrt{x}\, \hat{\jmath}$ and $\vec{r}(t) = t^2\, \hat{\imath} - t^3\, \hat{\jmath}$ for $0 \le t \le 1$.

$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = \langle t^4 \cdot (-t^3)^3, t^3 \cdot t \rangle \cdot \langle 2t, -3t^2 \rangle$

$= -2t^{14} - 3t^6$

$-\int_0^1 2t^{14} + 3t^6\, dt = -\left(\frac{2}{15} + \frac{3}{7}\right)$

$\begin{array}{r} \overset{3}{1}5 \\ \underline{7} \\ 105 \end{array}$

$= -\frac{(14+45)}{105}$

$= \boxed{\frac{-59}{105}}$

1) $\frac{17}{3}$. 2) $-\frac{59}{105}$.