

Practice Problems: Integrals in spherical coordinates

- 1) Evaluate $\iiint_E z \, dV$ where E lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.

$$\begin{aligned} & \int_0^{\pi/2} \int_0^{\pi/2} \int_1^2 p \cos \phi \underbrace{p^2 \sin \phi \, dp \, d\phi \, d\theta}_{dV} \\ &= \frac{\pi}{2} \left(\frac{16-1}{4} \right) \int_0^{\pi/2} \cos \phi \sin \phi \, d\phi = \boxed{\frac{15\pi}{16}} \end{aligned}$$

$\frac{1}{2} \frac{d}{d\phi} (\sin^2 \phi)$

- 2) Find the volume of the solid that lies above the cone $\phi = \pi/3$ and below the sphere $\rho = 4 \cos \phi$.

$\phi = \pi/3$ $\rho = 4 \cos \phi$

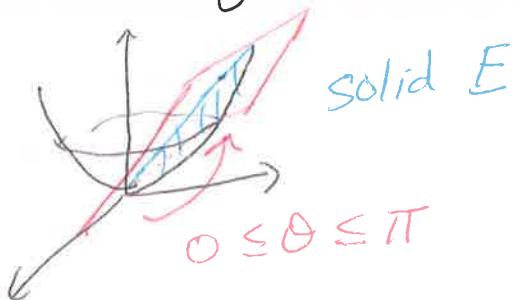
A diagram showing a sphere intersected by a cone. The cone is defined by the equation $\phi = \pi/3$. The sphere's radius is given as $4 \cos \phi$. The intersection creates a circular region on the sphere's surface.

$$\int_0^{2\pi} \int_0^{\pi/3} \int_0^{4 \cos \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

- 1) $15\pi/16$, 2) 10π

Problem in section

Q] $\iiint_E z \, dV = ?$ E: below $z=2y$
 above $z=x^2+y^2$



A] $0 \leq \theta \leq \pi$
 $\phi_{\text{plane}} \leq \phi \leq \pi/2$
 $0 \leq \rho \leq P_{\text{paraboloid}}$

• ϕ_{plane} : convert $z=2y$ into sphericals

$$\rho \cos \phi = 2 \rho \sin \phi \sin \theta$$

$$\Rightarrow \tan \phi = \frac{1}{2 \sin \theta} \Rightarrow \phi_{\text{plane}} = \tan^{-1} \left(\frac{1}{2 \sin \theta} \right)$$

got this
geometrically
in section

• $P_{\text{paraboloid}}$: convert $z=x^2+y^2$ into sphericals

$$\rho \cos \phi = (\rho \sin \phi)^2 \Rightarrow \cos \phi = \rho \sin^2 \phi$$

$$\Rightarrow P_{\text{paraboloid}} = \frac{\cos \phi}{\sin^2 \phi}$$

So $\int_0^{\pi} \int_{\tan^{-1}(1/(2 \sin \theta))}^{\pi/2} \int_0^{\cos \phi / \sin^2 \phi} (\rho \cos \phi) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = \boxed{\frac{5\pi}{6}}$

w/ computer