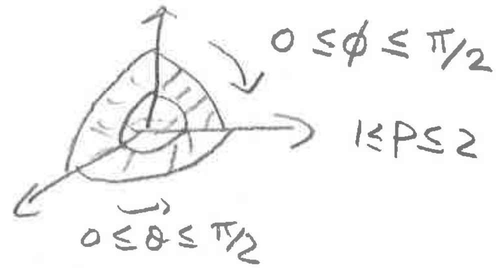


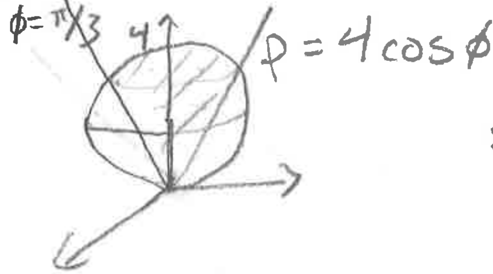
Practice Problems: Integrals in spherical coordinates

1) Evaluate $\int\int\int_E z\, dV$ where E lies between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$ in the first octant.

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 \underbrace{\rho\cos\phi}_{z} \cdot \underbrace{\rho^2\sin\phi\, d\rho\, d\phi\, d\theta}_{dV}$$

$$= \frac{\pi}{2}\left(\frac{16-1}{4}\right)\int_0^{\pi/2} \underbrace{\cos\phi\sin\phi}_{\frac{1}{2}\frac{d}{d\phi}(\sin^2\phi)}\, d\phi = \boxed{\frac{15\pi}{16}}$$

2) Find the volume of the solid that lies above the cone $\phi = \pi/3$ and below the sphere $\rho = 4\cos\phi$.

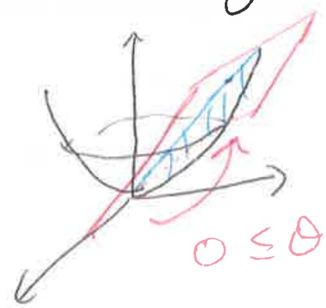


$$\int_0^{2\pi}\int_0^{\pi/3}\int_0^{4\cos\phi} \rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

1) $15\pi/16$, 2) 10π

Problem in section

Q] $\iiint_E z\, dV = ?$ E: below $z = 2y$, above $z = x^2 + y^2$



A] $0 \le \theta \le \pi$

$\phi_{plane} \le \phi \le \pi/2$

$0 \le \rho \le \rho_{paraboloid}$

- ϕ_{plane}: convert $z = 2y$ into sphericals

$$\cancel{\rho} \cos\phi = 2 \cancel{\rho} \sin\phi \sin\theta$$

$$\Rightarrow \tan\phi = \frac{1}{2\sin\theta} \Rightarrow \phi_{plane} = \tan^{-1}\left(\frac{1}{2\sin\theta}\right)$$

got this geometrically in section

- $\rho_{paraboloid}$: convert $z = x^2 + y^2$ into sphericals

$$\rho\cos\phi = (\rho\sin\phi)^2 \Rightarrow \cos\phi = \rho\sin^2\phi$$

$$\Rightarrow \rho_{paraboloid} = \frac{\cos\phi}{\sin^2\phi}$$

So $$\int_0^{\pi} \int_{\tan^{-1}\left(\frac{1}{2\sin\theta}\right)}^{\pi/2} \int_0^{\frac{\cos\phi}{\sin^2\phi}} (\rho\cos\phi)\, \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta = \boxed{\frac{5\pi}{6}}$$

w/ computer