Math 53 Discussion: Review

1) Use the transformation $u = x - y$, $v = x + y$ to evaluate $\int\int_R \frac{x-y}{x+y} dA$ where R is the square with vertices $(0,2)$, $(1,1)$, $(2,2)$ and $(1,3)$.

2) Find $\int\int\int_E yz \, dV$ where E lies above the plane $z = 0$, below the plane $z = y$, and inside the cylinder $x^2 + y^2 = 4$.

3) Describe the following surfaces in spherical coordinates: (i) the plane $z = a$, (ii) the sphere of radius a centered at $(0, 0, a)$, (iii) the cone $z = \sqrt{x^2 + y^2}$.

4) Set up $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y, z) = e^z\,\hat{\imath} + xz\,\hat{\jmath} + (x + y)\,\hat{k}$ along curve C given by $\vec{r}(t) = t^2\,\hat{\imath} + t^3\,\hat{\jmath} - t\,\hat{k}$ for $0 \leq t \leq 1$.

Over →

5) Use the Divergence theorem to calculate the flux of $\overrightarrow{F} = e^x \sin y\ \hat{\imath} + e^x \cos y\ \hat{\jmath} + yz^2\ \hat{k}$ across the surface $S =$ box bounded by the planes $x = 0, x = 1,\ y = 0, y = 1$ and $z = 0, z = 2$.

6) Find the area of the surface given by the part of the plane $x + 2y + 3z = 1$ that lies inside the cylinder $x^2 + y^2 = 3$.

7) Find the flux of $\overrightarrow{F} = \langle ze^{xy}, -3ze^{xy}, xy\rangle$ through the parallelogram with parametric equations $x = u + v, y = u - v$ and $z = 1 + 2u + v$ for $0 \le u \le 2$ and $0 \le v \le 1$, with upward orientation.

Answers: 1) The four sides are $y = x, y = 4 - x, y = 2 + x, y = 2 - x$. These map to the square $-2 \le u \le 0$ and $2 \le v \le 4$ in the uv-plane. Get $\int_2^4 \int_{-2}^0 \frac{u}{v}\frac{1}{2} du dv = -\ln 2$. 2) $\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_0^y yz\, dz dy dx$. Do z-integral, then convert to polars to get $\int_0^\pi \int_0^2 \frac{1}{2} r^3 (\sin^3\theta) r dr\, d\theta$. Use $\sin^3\theta = \sin\theta(1 - \cos^2\theta)$ to get $\frac{64}{15}$. 3) (i) $\rho = a\sec\phi,\ \rho = 2a\cos\phi,\ \phi = \pi/4$. 4) $\int_0^1 2te^{-t} - 3t^5 - t^2 - t^3\, dt$. 5) Since div $\overrightarrow{F} = 2yz$, we get $\int_0^2 \int_0^1 \int_0^1 2yz = 2$. 6) Graph of $z = \frac{1}{3}(1-x-2y)$ so $dS = \sqrt{1 + 1/9 + 4/9}\, dA$ and $\int\int_{x^2+y^2 \le 3} \sqrt{14}/3\, dA = \pi(\sqrt{3})^2\sqrt{14}/3 = \pi\sqrt{14}$. 7) Parametrize $\overrightarrow{r}(u, v) = (x(u, v), y(u, v), z(u, v))$. $\overrightarrow{r}_u = \langle 1, 1, 2\rangle$, $\overrightarrow{r}_v = \langle 1, -1, 1\rangle$, so $d\overrightarrow{S} = \pm\langle 3, 1, -2\rangle\, du dv$. Take $-$ so that z component is > 0, since we want upward orientation. Get $\int_0^1 \int_0^2 2(u^2 - v^2) du dv = 4$.

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~~Quiz on [redacted]: Review 16.6, 16.7, 16.9~~

1a) Find the centroid of a right circular cone with height h and base radius a. (Place the cone so that its base is in the xy-plane with center the origin and its axis along the positive z-axis.) b) Find the moment of inertia of the cone about its axis (z-axis).

2) Use polar coordinates to evaluate $\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (x^3 + xy^2)\, dy\, dx$.

3) Rewrite the integral $\int_{-1}^{1} \int_{x^2}^{1} \int_0^{1-y} f(x, y, z)\, dz\, dy\, dx$ as an iterated integral in order $dx\, dy\, dz$.

4) Evaluate $\int_C xy\, dx + y^2\, dy + yz\, dz$ where C is the line segment from $(1, 0, -1)$ to $(3, 4, 2)$.

5) Show that $\vec{F}(x, y, z) = \sin y\ \hat{\imath} + x \cos y\ \hat{\jmath} - \sin z\ \hat{k}$ is conservative, and find a potential function f so that $\vec{F} = \nabla f$.

6) If f is a harmonic function, that is, $\nabla^2 f = 0$, show that the line integral $\int_C f_y\, dx - f_x\, dy$ is independent of path C in any simple region D.

7) Evaluate $\int\int_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = xz\ \hat{\imath} - 2y\ \hat{\jmath} + 3x\ \hat{k}$ and S is the sphere $x^2 + y^2 + z^2 = 4$ with outward orientation.

Answers: 1a) $(0, 0, h/4)$, b) $\pi a^4 h/10$. 2) $486/5$. 3) $\int_0^1 \int_0^{1-z} \int_{-\sqrt{y}}^{\sqrt{y}}$. 4) $110/3$. 5) $x \sin y + \cos z + k$. 6) By Green's theorem, if C is a simple closed curve then the line integral is zero. 7) $-64\pi/3$.