

## Math 53 Discussion: Review problems for Midterm 1

- 1) Find the area of the the region lying inside both  $r = 2 \sin \theta$  and  $r = \sin \theta + \cos \theta$ .
- 2) Find the length of the curve  $x(t) = 3t^2$ ,  $y(t) = 2t^3$  for  $0 \leq t \leq 2$ .
- 3) a) Find a vector perpendicular to the plane through the points  $A(1,0,0)$ ,  $B(2,0,-1)$  and  $C(1,4,3)$ . b) Find the area of the triangle  $ABC$ .
- 4) Find an equation of the plane through  $(3, -1, 1)$ ,  $(4, 0, 2)$  and  $(6, 3, 1)$ .
- 5) A surface consists of all points  $P$  such that the distance from  $P$  to the plane  $y = 1$  is twice the distance from  $P$  to the point  $(0, -1, 0)$ . Find the equation for the surface and identify.
- 6) Find the point where the line  $\vec{r}(t) = \langle 2 - t, 1 + 3t, 4t \rangle$  intersects the plane  $2x - y + z = 2$ .
- 7) A particle moves with position function  $\vec{r}(t) = t \ln t \hat{i} + t \hat{j} + e^{-t} \hat{k}$ . Find the velocity, speed and acceleration of the particle.
- 8) An athlete throws a shot at an angle of  $45^\circ$  to the horizontal at an initial speed of 43 ft/s. The ball leaves his hand 7 ft above the ground. Assuming acceleration comes from gravity only, find the position vector describing the ball's trajectory.
- 9) Question 49 from Chapter 14 review of the textbook (reading a contour plot of hurricane wind speed.)
- 10) Use polar coordinates to find the limit: 
$$\lim_{(x,y) \rightarrow (0,0)} \frac{e^{-x^2-y^2} - 1}{x^2 + y^2}.$$
- 11) Find the equation of the normal line and tangent plane to the surface  $z = 3x^2 - y^2 + 2x$  at  $(1, -2, 1)$ .
- 12) Find the linear approximation of  $f(x, y, z) = x^3 \sqrt{y^2 + z^2}$  at  $(2, 3, 4)$ .
- 13) Use the Chain Rule to find  $\frac{du}{dp}$  where  $u(x, y) = x^2 y^3$ ,  $x(p) = p + 3p^2$ ,  $y(p) = pe^p$ .
- 14) Find the directional derivative of  $f(x, y) = x^2 e^{-y}$  in the direction towards  $(2, -3)$  from the point  $(-2, 0)$ .
- 15) Find the absolute maximum and minima of  $f(x, y) = 4xy^2 - x^2 y^2 - xy^3$  on  $D$ , the closed triangular region with vertices at  $(0, 0)$ ,  $(0, 6)$  and  $(6, 0)$ .

Answers: 1)  $\frac{1}{2}(\pi - 1)$ , 2)  $2(5\sqrt{5} - 1)$ , 3)  $\langle 4, -3, 4 \rangle$ ,  $\frac{\sqrt{41}}{2}$ , 4)  $-4x + 3y + z = -14$ , 5)  $(1, 4, 4)$ ,  
6)  $4x^2 + 3y^2 + 10y + 4z^2 + 3 = 0$ , ellipsoid. 7)  $\vec{r}'(t) = (1 + \ln t) \hat{\mathbf{i}} + \hat{\mathbf{j}} - e^{-t} \hat{\mathbf{k}}$ ,  $|\vec{v}(t)| = \sqrt{(1 + \ln t)^2 + 1 + e^{-2t}}$ ,  
 $\vec{a}(t) = \frac{1}{t} \hat{\mathbf{i}} + e^{-t} \hat{\mathbf{k}}$ , 8)  $\vec{r}(t) = \frac{43}{\sqrt{2}}t \hat{\mathbf{i}} + \left(\frac{43}{\sqrt{2}}t - \frac{1}{2}gt^2 + 7\right) \hat{\mathbf{j}}$ , 9)  $\approx \frac{5}{8}$ , 10)  $-1$ . 11) plane:  $z = 8x +$   
 $4y + 1$ , normal line:  $x = 1 + 8t, y = -2 + 4t, z = 1 - t$ , 12)  $f(x, y, z) \approx 60x + (24/5)y + (32/5)z - 120$ ,  
13)  $2xy^3(1 + 6p) + 3x^2y^2(e^p + pe^p)$ , 14)  $-\frac{4}{5}$ , 15) max at  $(1, 2)$  and min at  $(2, 4)$