

Math 53 Discussion: Practice with line integrals

1) Determine if $\vec{F}(x, y) = (\ln y + 2xy^3)\hat{i} + (3x^2y^2 + x/y)\hat{j}$ is conservative. If so, find a scalar potential function f , i.e. such that $\vec{F} = \nabla f$.

2) Explain why the following holds: Suppose a vector field $\vec{F}(x, y)$ is perpendicular to the tangent vector $\vec{r}'(t)$ to a curve C , at each point $(x(t), y(t))$ on the curve. Then $\int_C \vec{F} \cdot d\vec{r} = 0$.

Extra line integral practice

3) Evaluate $\int_C(xy + \ln x)dy$ where C is the arc of the parabola $y = x^2$ from $(1, 1)$ to $(3, 9)$.

4) The position of an object with mass m at time t is $\vec{r}(t) = at^2\hat{i} + bt^3\hat{j}$, for $0 \leq t \leq 1$.
(a) Find the force acting on the object at time t . (b) What is the work done by the force during $0 \leq t \leq 1$?

Answers: 1) Yes, $x \ln y + x^2y^3 + C$. 2) dot product in integral is zero along C . 3) $\frac{464}{5} + 9 \ln 3$.
4) a) Use $\vec{F} = m\vec{a}$ to get $2ma\hat{i} + 6bmt\hat{j}$, b) $m(2a^2 + \frac{9}{2}b^2)$.