Math 53 Discussion: Max/Min, gradient practice problems (2 sides)

1) [Worksheet 14] Find and classify the critical points of the following functions:

(a) $f(x, y) = x^5 + y^4 - 5x - 32y + 81$
(b) $f(x, y) = x^3 + y^3 + 3xy - 27$
(c) $f(x, y) = x^2y + 3xy - 3x^2 - 4x + 2y + 1$

2) A cardboard box without a lid is to have a volume of 32 cm^3. Find the dimensions that minimize the amount of cardboard used.

3) Are there any points on the hyperboloid $x^2 - y^2 - z^2 = 1$ where the tangent plane is parallel to the plane $z = x + y$?

4) Show that $\nabla(uv) = u\nabla v + v\nabla u$ for u, v differentiable functions of x and y.

5) Suppose that the directional derivatives of $f(x, y)$ are known at a given point in two non-parallel directions given by unit vectors $\hat{u}$ and $\hat{v}$. Is it possible to find ∇f at this point? If so, how would you do it?

Answers. 1a) local min at $(1, 2)$ and saddle at $(-1, 2)$, b) local max at $(-1, -1)$ and saddle at $(0, 0)$, c) saddle points at $(-1, -2)$ and $(-2, 8)$. 2) $(l, h, w) = (4, 2, 4)$. 3) No. 4) Write out in components and use the product rule. 5) Yes: if we're given $D_{\hat{u}} f = f_x u_1 + f_y u_2 = a_1$ and $D_{\hat{v}} f = f_x v_1 + f_y v_2 = a_2$ then

$$\begin{pmatrix} f_x \\ f_y \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \\ v_1 & v_2 \end{pmatrix}^{-1} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$