Math 53 Discussion–vector functions, lines and planes

Quiz Wednesday on 12.5–12.6 and 13.1–13.2

1) Find the tangent vector function $\overrightarrow{r}'(t)$ for the curve $\overrightarrow{r}(t) = \langle t^2, \cos 2t, -te^{-t}\rangle$. Find the equation of the tangent line to the curve at $t = 0$.

2) [13.4, #13] Find the velocity, acceleration, and speed of a particle with the position function $\overrightarrow{r}(t) = e^t(\cos t\ \hat{\mathbf{i}} + \sin t\ \hat{\mathbf{j}} + t\ \hat{\mathbf{k}})$.

3) [13.4, #15] Find the velocity and position vectors of a particle that has acceleration $\overrightarrow{a}(t) = \hat{\mathbf{i}} + 2\ \hat{\mathbf{j}}$, $\overrightarrow{v}(0) = \hat{\mathbf{k}}$, $\overrightarrow{r}(0) = \hat{\mathbf{i}}$.

4) [13.4, #45] The position function of a spaceship is

$$\overrightarrow{r}(t) = (3+t)\hat{i} + (2+\ln t)\hat{j} + \left(7 - \frac{4}{t^2+1}\right)\hat{k}$$

and the coordinates of a space station are $(6, 4, 9)$. The captain wants the spaceship to coast into the space station. When should the engines be turned off?

Additional practice with lines and planes.

5) [12.5, #78] Find the distance between the skew lines with parametric equations

$$x = 1+t, y = 1+6t, z = 2t$$

$$x = 1+2s, y = 5+15s, z = -2+6s$$

6) [12.5, #37] Find the equation of the plane containing the following point and line: the point is $(-1, 2, 1)$ and the line is given by the intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.

Answers: 1) $\overrightarrow{r}'(t) =< 2t, -2\sin 2t, e^{-t}(-1+t) >$, $< 0, 1, -t >$. 2) $\overrightarrow{v}(t) = e^t < \cos t - \sin t, \sin t + \cos t, t+1 >$, $\overrightarrow{a}(t) = e^t < -2\sin t, 2\cos t, t+2 >$, $|\overrightarrow{v}(t)| = e^t\sqrt{t^2+2t+3}$. 3) $\overrightarrow{r}(t) = (\frac{1}{2}t^2+1)\,\hat{\mathbf{i}} + t^2\,\hat{\mathbf{j}} + t\,\hat{\mathbf{k}}$, $\overrightarrow{v}(t) = t\,\hat{\mathbf{i}} + 2t\,\hat{\mathbf{j}} + \hat{\mathbf{k}}$. 4) $t = 1$. 5) 2. Hint: skew lines lie on parallel planes. You can think of these lines lying on two floors of a house. You want to find the distance between the two floors. Use scalar projection. 6) $x - 2y + 4z = -1$. Hint: To find a plane containing the point and line, we need to get a normal vector. To get a normal, we need to take a cross product of two vectors lying in the plane. You can take one of these vectors to be the direction of the line of intersection. The other vector could be the vector from our distinguished point $(-1, 2, 1)$ to some point of your choosing on the line of intersection.