Math 53 Discussion–Practice Problems [2 sides]: Dot product, cross product, area of a parallelogram, scalar triple product, volume of parallelepiped

1) [#43, 12.3] Find the scalar and vector projections of $\overrightarrow{b} = \left\langle 0, 1, \frac{1}{2} \right\rangle$ onto $\overrightarrow{a} = \langle 2, -1, 4 \rangle$.

2) Find the area of the triangle with vertices $P(1, 2, 3)$, $Q(1, 3, 6)$ and $R(3, 5, 6)$.

3) From the previous question we have points P, Q, and R. Consider an additional fourth point $S(1, 4, 2)$. Find $\overrightarrow{PS}$. Find the volume of the parallelepiped spanned by $\overrightarrow{PQ}, \overrightarrow{PR}$, and $\overrightarrow{PS}$. (*Hint: Scalar triple product or* 3×3 *determinant.*)

4) [Worksheet 4 from online, Q4] a) Suppose that one side of a triangle forms the diameter of a circle and the vertex opposite this side lies on a circle. Use the dot product to prove that this is a right triangle. b) Now do the same in $\mathbb{R}^3$.

5) Prove the addition trig formulas for $\cos(\theta_2 - \theta_1)$ and $\sin(\theta_2 - \theta_1)$ using the dot product and cross product.

Answers: 1) $1/\sqrt{21}$, $\frac{1}{21} < 2, -1, 4 >$. 2) $\sqrt{19}$. 3) 14. 4) Given the triangle inscribed in the circle, draw the three radii from the center of the circle out to the three vertices of the triangle. Label these vectors $\overrightarrow{a}$ and $\overrightarrow{b}$. Then use vector addition. The same proof applies in $\mathbb{R}^3$. 5) Draw unit vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ at angles θ_1 and θ_2 respectively above the positive x-axis. Use the two different expressions for dot product to get the first identity, and likewise with cross product to get the second identity.