Math 53 Discussion: Review problems for Midterm 1

1) Find the area of the the region lying inside both $r = 2\sin\theta$ and $r = \sin\theta + \cos\theta$.

2) Find the length of $x = 3t^2$, $y = 2t^3$ for $0 \leq t \leq 2$.

3) a) Find a vector perpendicular to the plane through the points $A(1,0,0)$, $B(2,0,-1)$ and $C(1,4,3)$. b) Find the area of the triangle ABC.

4) Find an equation of the plane through $(3,-1,1)$, $(4,0,2)$ and $(6,3,1)$.

5) Find the point in which the line $2-t, 1+3t, 4t$ intersects the plane $2x - y + z = 2$.

6) A particle moves with position function $\overrightarrow{r}(t) = t\ln t\hat{i} + t\hat{j} + e^{-t}\hat{k}$. Find the velocity, speed and acceleration of the particle.

7) An athlete throws a shot at an angle of 45° to the horizontal at an initial speed of 43 ft/s. The ball leaves his hand 7 ft above the ground. Assuming acceleration comes from gravity only, find the position vector describing the ball's trajectory.

8) Question 49 from Chapter 14 review of the textbook (reading a contour plot of hurricane wind speed.)

9) Find the equation of the tangent plane and normal line to the surface $z = 3x^2 - y^2 + 2x$ at $(1,-2,1)$.

10) Find the linear approximation of $x^3\sqrt{y^2+z^2}$ at $(2,3,4)$.

11) Use the Chain Rule to find du/dp where $u = x^2y^3, x = p + 3p^2, y = pe^p$.

12) Find the directional derivative of $f = x^2e^{-y}$ in the direction towards $(2,-3)$ from the point $(-2,0)$.

13) Find the absolute maximum and minima of $f(x,y) = 4xy^2 - x^2y^2 - xy^3$ on D, the closed triangular region with vertices at $(0,0), (0,6)$ and $(6,0)$.

14) Use Lagrange multipliers to find the maximum and minimum values of $f(x,y) = x^2y$ subject to $x^2 + y^2 = 1$.

Answers: 1) $1/2(\pi - 1)$, 2) $2(5\sqrt{5} - 1)$, 3) $\langle 4, -3, 4\rangle$, $\sqrt{41}/2$, 4) $-4x + 3y + z = -14$, 5) $(1, 4, 4)$, 6) $\overrightarrow{r}'(t) = (1 + \ln t)\hat{i} + \hat{j} - e^{-t}\hat{k}$, $|\overrightarrow{v}(t)| = \sqrt{(1 + \ln t)^2 + 1 + e^{-2t}}$, $\overrightarrow{a}(t) = \frac{1}{t}\hat{i} + e^{-t}\hat{k}$, 7) $\overrightarrow{r}(t) = \frac{43t}{\sqrt{2}}\hat{i} + \left(\frac{43}{\sqrt{2}}t - \frac{1}{2}gt^2 + 7\right)\hat{j}$, 8) $\approx 5/8$, 9) $z = 8x + 4y + 1$, $x = 1 + 8t, y = -2 + 4t, z = 1 - t$, 10) $f(x, y, z) \approx 60x + (24/5)y + (32/5)z - 120$, 11) $2xy^3(1 + 6p) + 3x^2y^2(e^p + pe^p)$, 12) $-4/5$, 13) max at $(1, 2)$ and min at $(2, 4)$, 14) max value $\frac{2}{3\sqrt{3}}$ and min value $\frac{-2}{3\sqrt{3}}$