Math 53 Discussion

Practice Problems: Sections 14.3-14.5. Equation of the tangent plane, linear approximations, Clairaut's Theorem, the Chain Rule

1) Verify that $f(x,y) = e^x \cos(xy)$ is differentiable at $(0,0)$ and find the linearization of $f(x,y)$ at $(0,0)$.

2) Clairaut's Theorem says that if f_{xy} and f_{yx} are continuous on some disk containing a point, then they are equal at that point. Verify Clairaut's theorem holds with $f(x,y) = e^x \cos(xy)$ above, for all (x,y).

3) Use linear approximation to estimate the amount of tin in a closed cylindrical tin can with radius 4 cm and height 12 cm, if the tin is 0.04 cm thick on all sides. [*I.e. you want to find a change ΔV in volume.*]

4) Find $\dfrac{dz}{dt}$ where $z = \sqrt{1 + x^2 + y^2}$ and $x = \ln t$, $y = \cos t$.

5) Use the example in class of a box with dimensions ℓ, w, h changing in time: at time t_1, $\ell = 1$ m, $w = h = 2$ m and the rate of change of ℓ, w are 2 m/s while for h it is -3 m/s. Find the rates at which (a) the surface area and (b) the length of the main box diagonal are changing at time t_1.