Math 53 Discussion

Practice Problems: Cross product, area of a parallelogram, scalar triple product, volume of parallelepiped

1) Find $\langle 6, 0, -2\rangle \times \langle 0, 8, 0\rangle$.

2) Find the area of the triangle with vertices $P(1, 2, 3)$, $Q(1, 3, 6)$ and $R(3, 5, 6)$. (*Hint: First find* $\overrightarrow{PQ}$ *and* $\overrightarrow{PR}$*. Then use the cross product.*)

3) Geometrically, why is $(\overrightarrow{a} \times \overrightarrow{b}) \cdot \overrightarrow{a} = 0$ for all vectors $\overrightarrow{a}$, $\overrightarrow{b}$ in $\mathbb{R}^3$?

4) Find two unit vectors orthogonal to both $\overrightarrow{a} = \langle 3, 2, 1\rangle$ and $\overrightarrow{b} = \langle -1, 1, 0\rangle$. What other vectors are orthogonal to both $\overrightarrow{a}$ and $\overrightarrow{b}$?

5) From question 2) we have points P, Q, and R. Consider an additional fourth point $S(1, 4, 2)$. Find $\overrightarrow{PS}$. Find the volume of the parallelepiped spanned by $\overrightarrow{PQ}, \overrightarrow{PR}$, and $\overrightarrow{PS}$. (*Hint: Scalar triple product or* 3×3 *determinant.*)