Math 53 Discussion

Quiz on Monday: Sections 14.4–14.6: linear approximation, Chain rule, directional derivative, gradient

Practice Problems: Sections 14.5–14.6: Second order partial derivatives, the gradient and its physical meaning

1) Find ∇f for $f(x, y, z) = x^2yz - xyz^3$. Evaluate the gradient at the point $P(2, -1, 1)$. Find the rate of change of f at P in the direction of the vector $\overrightarrow{u} = \langle 0, 4/5, -3/5 \rangle$.

2) Suppose that over a certain region of space the electric potential is given by $V(x, y, z) = 5x^2 - 3xy + xyz$. (a) Find the rate of change of the potential at $P(3, 4, 5)$ in the direction of the vector $\overrightarrow{v} = \hat{i} + \hat{j} - \hat{k}$. (b) In which direction does V change most rapidly at P? (c) What is the maximum rate of change at P? (You don't need to simplify your expression in (c).)

3) Find the equation of the tangent plane to the surface $x + y + z = e^{xyz}$ at the point $(0, 0, 1)$.

4) Are there any points on the hyperboloid $x^2 - y^2 - z^2 = 1$ where the tangent plane is parallel to the plane $z = x + y$?

5) Show that $\nabla(uv) = u\nabla v + v\nabla u$ for u, v differentiable functions of x and y.

6*) Suppose that the directional derivatives of $f(x, y)$ are known at a given point in two non-parallel directions given by unit vectors $\hat{u}$ and $\hat{v}$. Is it possible to find ∇f at this point? If so, how would you do it?

Extra chain rule practice:
[These 3 won't be discussed in section but answers are posted at the bottom.]

1) Use a tree diagram to write out the Chain Rule for $u = f(x, y)$ and $x = x(r, s, t)$, $y = y(r, s, t)$.

2) Let $w = xy + yz + zx$, $x = r\cos\theta$, $y = r\sin\theta$, $z = r\theta$. Use the Chain Rule to find $\partial w/\partial r$ and $\partial w/\partial\theta$ when $r = 2$ and $\theta = \pi/2$.

3) The temperature at a point (x, y) is $T(x, y)$, measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x = \sqrt{1+t}$, $y = 2 + \frac{1}{3}t$, where x and y are measured in centimeters. The temperature function satisfies $T_x(2, 3) = 4$ and $T_y(2, 3) = 3$. How fast is the temperature rising on the bug's path after 3 seconds?

1) u on top, connected to x and y in the 2nd row, each connected to r, s, and t in the 3rd row. 2) $2\pi, -2\pi$. 3) 2