Topics since Midterm 2: Chapter 15: Double integrals (rectangular and polar coordinates), applications of double integrals (average value, center of mass, moment of inertia, probability), change of variables, triple integrals (rectangular, cylindrical, spherical), applications of triple integrals.

Chapter 16: line integrals of a scalar function, line integrals of a vector (work, flux), conservative vector fields and fundamental theorem of calculus, Green's theorem (2 versions - for work, for flux). Parametric surfaces: surface integral of a scalar function, surface integral of a vector (flux), Divergence theorem, Stokes' theorem. Know $d\overrightarrow{S} = \hat{n}\, dS$ for a plane, sphere, cylinder, graph of a function.

Practice Problems

1) Let P be a point at a distance d from the center of a circle of radius r, where $d < r$. The curve traced out by P as the circle rolls along a straight line is called a trochoid. (The cycloid is the case when $d = r$.) Show that the parametric equations of the trochoid are $x = r\theta - d\sin\theta$, $y = r - d\cos\theta$, where θ is the same parameter as for the cycloid.

2) Find the area of the region that lies inside the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$.

3) Find the equation of the plane that passes through the point $(1, 5, 1)$ and is perpendicular to the planes $2x + y - 2z = 2$ and $x + 3z = 4$.

4) Let $u(t) = \overrightarrow{r}(t) \cdot [\overrightarrow{r}'(t) \times \overrightarrow{r}''(t)]$. Show that $u'(t) = \overrightarrow{r}(t) \cdot [\overrightarrow{r}'(t) \times \overrightarrow{r}'''(t)]$.

5) Verify the linear approximation $\sqrt{y + \cos^2 x} \approx 1 + \frac{1}{2}y$ at $(0, 0)$.

6) Let $f(x, y) = 2x^2 + 3y$ and $\overrightarrow{v} = \langle 1, 2\rangle$. Find the directional derivative $D_{\hat{u}}f$ at $(1, 1)$, where $\hat{u}$ is in the direction of $\overrightarrow{v}$.

7) Find the nature of the critical points of $f(x, y) = x^3 + y^3 + 3xy - 27$ on $\mathbb{R}^2$. Are there global maxima, minima?

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8) [#31, 15.9] Let E be the solid bounded above by the sphere $x^2+y^2+z^2=z$ and below by the cone $z=\sqrt{x^2+y^2}$, with constant density K. (a) Find the center of mass of this solid. (b) Find the moment of inertia about the z-axis for the solid.

9) Let $\overrightarrow{F}=\left\langle 2x+\cos(y^2), -(y+e^{\sqrt{x}})\right\rangle$. Find the flux of $\overrightarrow{F}$ through the positively oriented boundary of the region enclosed by $y=x^2$ and $x=y^2$.

10) [Example 2, 16.9] Evaluate $\int\int_S \overrightarrow{F}\cdot d\overrightarrow{S}$ where $\overrightarrow{F}(x,y,z)=xy\,\hat{\mathbf{i}}+(y^2+e^{xz^2})\,\hat{\mathbf{j}}+\sin(xy)\,\hat{\mathbf{k}}$ and S is the surface of the region E bounded by the parabolic cylinder $z=1-x^2$ and the planes $z=0, y=0,\ y+z=2$.

11) [#15, 16.8] Verify Stokes' theorem is true for $\overrightarrow{F}(x,y,z)=\langle y,z,x\rangle$ where S is the hemisphere $x^2+y^2+z^2=1$ for $y\geq 0$, oriented in the direction of the positive y-axis.

Answers: 2) π. 3) $3x-8y-z=-38$. 6) $2\sqrt{5}$. 7) Saddle at $(0,0)$, local max at $(-1,-1)$. No global max/min. 8a) $(0,0,7/12)$, b) $11K\pi/960$. 9) $1/3$. 10) $\frac{184}{35}$. 11) $-\pi$.